Determination of intensity factors at an arched crack tip by the method of caustics

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Perpendicular sections of caustic surfaces reflected by the constrained zone around the tip of an arched crack in a perspex plate under plane tension, were measured and analysed. The experiments corroborated the theory, the Theocaris' method of Caustics and also show that the intensity factors are functions of the curvature of the crack and the subtended angle.

Notation

c constant depending on optical properties of specimen

C $=$dtc

d distance of specimen from screen

d_1 distance of specimen from focus

K^+ $=K_i - iK_{ii}$

K_i opening mode

K_{ii} sliding mode

q constant determined by the fixed value of R_c and γ

r $=|z|$

r_o $=$radius of the initial curve$=\left(\dfrac{3CK^+}{2\sqrt{2\pi}}\right)^{2/5}$

R $=|W|$

R_c radius of curvature

Re real part of

t specimen thickness

W $=X+iY$

z $=x+iy$

$\bar{z}$ $=x-iy$

β 1/2 subtended angle

γ inclination of chord relative to applied loading

δ $=EC/r_o$

θ $=$Arc z

λ $=$Arg. W

μ $=-K_{ii}/K_i$

ρ magnification ratio

σ applied loading (stress)

Φ $=\theta-2\omega$

ω $=\tan^{-1}\mu$

Introduction

This paper summarises work planned with the object of investigating whether data relating to stress intensity applied to the case of a circular arc crack agree with the general theory, and the predicted expressions, for the determination of stress intensity factors.

The theoretical work is concerned with linear elastic fracture mechanics and is based on a set of equations derived by Westergaard.[1, 5] These were developed for the three possible modes of displacement of the crack surfaces; the opening mode, the sliding mode and the tearing mode. Any type of crack deformation can be represented as a combination of the above three modes. This paper deals with internal circular-arc cracks in thin plates for which a plane-stress analysis can be used without significant loss of accuracy. The situation considered is that of a combination of *opening* and *sliding* mode only.

Mathematicians and experimenters have attempted to deduce expressions representing the stress field for cracks of various geometrical forms. Solutions include (a) analytical,[1, 2, 5, 6] based on Westergaard's combination of equations for the three modes, (b) numerical, (c) experimental, based on optical methods. Some of these methods have not been very successful in evaluating the individual stresses σ_x, σ_y, σ_{xy}, independently, point by point. One of the first and most successful experimental techniques for the determination of the stress field around a crack was the photoelastic method. Photoelasticity[2, 3, 4] provides information about the variation of the difference between the principal stresses at points in the field; however this is not accurate enough for the determination of the differences in the vicinity of the crack *tip* where an elastic singularity complicates and confuses the stress-optical pattern due to the rapid change of stress near the crack tip. The crucial region, where the high stress gradients occur, is very small; the contour lines (representing equal values of principal stress difference) become very tightly packed and ill-defined. Thus it was necessary to extrapolate from known values as near to the crack tip as possible.

The method of '*Caustics*' developed recently by Theocaris,[7, 8] provides a way of determining the sum of the principal stresses at a point, instead of the difference. In this paper the method of caustics is briefly discussed and is used to evaluate the stress intensity factors in the vicinity of the tips of circular cracks.

The method of caustics

Light rays emitted from a given source, after reflection or refraction by a given curved surface, in general do not focus at one point. Consecutive rays intersect at points lying on a caustic curved surface (Greek καυστικός, 'burning') so called because along it the intersection of consecutive rays forms an envelope of maximal concentration of luminous and heating effects. Such rays are generally tangents to the *caustic* which is observed as a bright curve when projected onto a screen (Figs 7, 8, 9).

By varying the curvature of the reflecting surface, or moving the source of light, a variety of patterns can be obtained. Huygens, Bernoulli and others studied the geometry of those patterns, the *caustics*, with reference to the reflecting surface.

An inverse procedure has been discussed[11] and applied elsewhere[7, 8, 9, 10]; now the geometry of the caustic is used to determine the topography of the surface, its stress field and the intensity factors acting upon it.

The method of caustics as used in the experiment of this paper employs a transparent slitted plane specimen made from an isotropic material which is loaded under conditions of plane stress and illuminated by a monochromatic beam of light. The light is partially reflected from the front face of the specimen and is also reflected from the rear face after having been twice refracted.

The variation in curvature of the lateral faces close to the crack tip, together with the variation of the refractive index (Maxwell-Neumann law), causes the reflected and refracted rays to deviate and concentrate along a three-dimensional surface which is an envelope formed by the intersection of rays refracted and reflected from the surfaces around the crack tip; its two-dimensional section can be recorded on a screen perpendicular to the axis of projection, magnified and measured.

The bright curve shown in Figs 7, 8, 9 and 10(i), is the section of the *caustic* surface. The surrounding fringes are the result of the interference of the light rays reflected from the back and front faces of the transparent specimen either cancelling or reinforcing each other. The fringe patterns are a clue to the evaluation of stresses,[12] but in the case of an opaque specimen (e.g. metal) fringes do *not* appear around the caustic.

The caustic curve

Let P(x, y) be a point on the specimen (Fig 1). The reflected rays will map P on point Q(X, Y) on the screen. The complex representation of Q, with respect to the projection of the x, y axes on the screen, is given by,

$$W = z + C \operatorname{grad} (\sigma_1 + \sigma_2) \qquad (1)$$

where $z = x + iy$, $W = X + iY$, and C is a constant depending on the optical properties of the specimen material.[7, 9] $\sigma_1 + \sigma_2$ is given by the Westergaard relation[1, 5] as,

$$\sigma_1 + \sigma_2 = 4\,\mathrm{Re}\left(\frac{K^+}{(2\pi z)^{1/2}}\right)$$

where $K^+ = K_i - iK_{ii}$ and $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$. Substituting the value of $\sigma_1 + \sigma_2$ into (1) we get

$$W = z + \frac{CK^+}{2\pi}\bar{z}^{-3/2} \qquad (2)$$

Making use now of the mathematical condition for the points Q on the screen to form a continuous curve (zeroing of the Jacobian of the transformation from the (x, y) axes to the (X, Y) axes), yields,

$$|z| = \left(\frac{3CK^+}{2\sqrt{2\pi}}\right)^{2/5} = r_o \qquad (3)$$

called the equation of the initial curve.
(2) and (3) now form a system which represents the caustic curve.

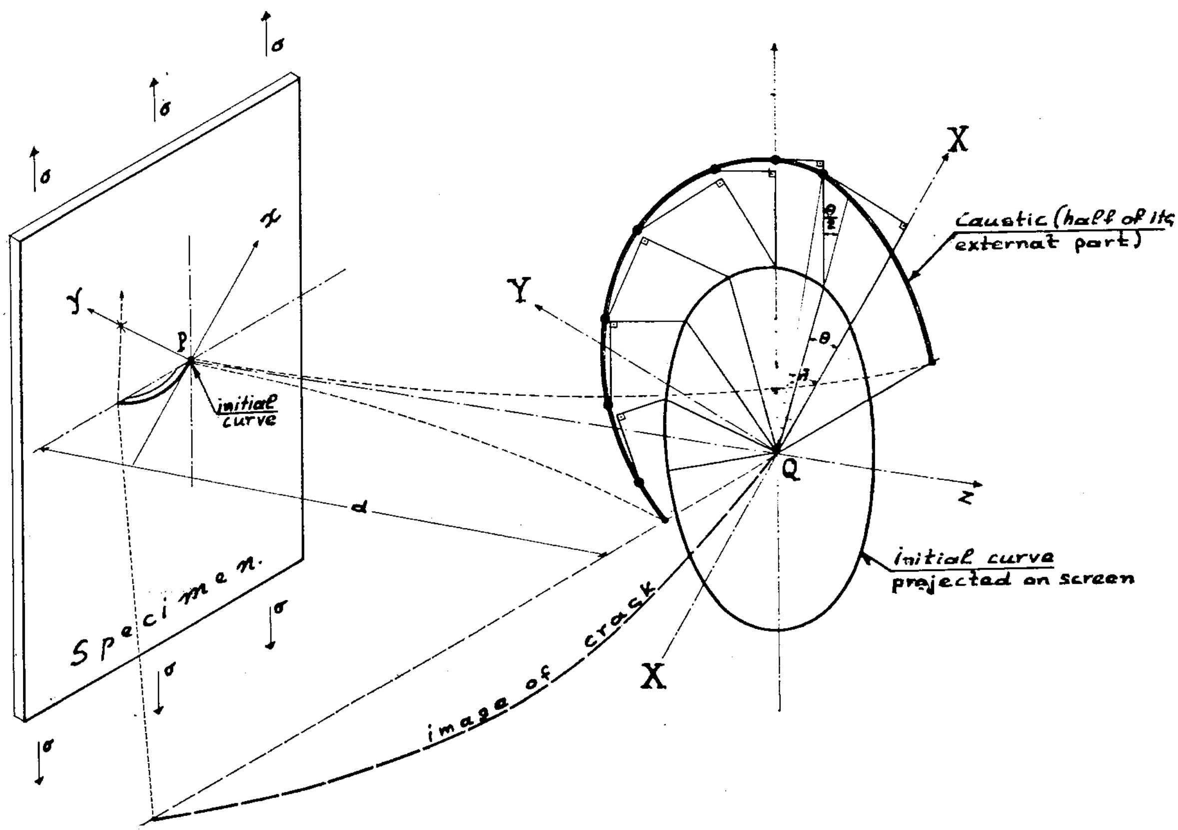

Fig 1. Schematic of the formation of a caustic by the tip of a circular arc crack. The geometrical construction of the curve is explained elsewhere[10].

Choosing axes tangential and normal to the crack tip, letting

$$\frac{K_{ii}}{K_i}=\mu=\tan\omega \tag{4}$$

and combining (2) and (3), the equation of the caustic can be written in polar form as,

$$W=Re^{i\lambda}=\left\{e^{i\Theta}+\frac{2}{3}e^{i\left(\frac{3\Theta}{2}+\omega\right)}\right\}r_c \tag{5}$$

and in parametric form as:

$$X=r_o\left\{\cos\Theta+\frac{2}{3}\frac{1}{(1+\mu)^{1/2}}\cos\frac{3\Theta}{2}-\frac{2}{3}\frac{\mu}{(1+\mu)^{1/2}}\sin\frac{3\Theta}{2}\right\}$$

$$Y=r_o\left\{\sin\Theta+\frac{2}{3}\frac{1}{(1+\mu)^{1/2}}\sin\frac{3\Theta}{2}+\frac{2}{3}\frac{\mu}{(1+\mu)^{1/2}}\cos\frac{3\Theta}{2}\right\} \tag{6}$$

Now considering new axes, which are obtained from the old system (the one tangential to the crack tip) by rotation through an angle $2(\pi-\omega)$,

$$W'=W\cdot e^{-i2(\pi-\omega)}$$

the parametric equations of which are:

$$\left.\begin{aligned}X'&=r_o\cos\Phi+\frac{2}{3}r_o\cos\frac{3\Phi}{2}\\ Y'&=r_o\sin\Phi+\frac{2}{3}r_o\sin\frac{3\Phi}{2}\end{aligned}\right|\quad \text{where } \Theta+2\omega=\Phi \tag{7}$$

The above equations describe a curve (an epicycloid) (cf. other work[9, 11]), which is symmetrical about the new X′ axis. The X′ axis is rotated in such a way that it forms an angle -2ω with the tangent at the crack tip. As seen from equations (4) and (6), the ratio of the opening-mode and sliding-mode stress intensity factors actually determines the inclination of the axis of symmetry of the caustic, with respect to the tangent at the crack tip. Furthermore r_o depends on $|K^+|=K_i-iK_{ii}$, thus the *size* of the caustic is dependent on $(K_i^2+K_{ii}^2)$.[9, 10]

Circular arc crack

For the case of circular arc cracks the factors determining the values of K_i and K_{ii} are:

—the applied loading σ,
—the curvature $1/R_c$,
—the subtended angle 2β,
—the inclination γ of the chord relative to the direction of the applied loading, *and*—the inclination of the X axis to the tangent at the crack tip relative to the direction of the applied loading.

The last factor is directly related to all other previous factors, because of the particular geometry of the circular arc. In this paper the simple case is considered of cracks of constant radius, whose chord is either *parallel*, or *perpendicular* to the axis of applied loading (Figs 7, 8, 9, 10); thus $K_i=qf_1(\sigma,\beta)$, $K_{ii}=qf_2(\sigma,\beta)$, where q is a constant determined by the fixed value of R_c and γ.

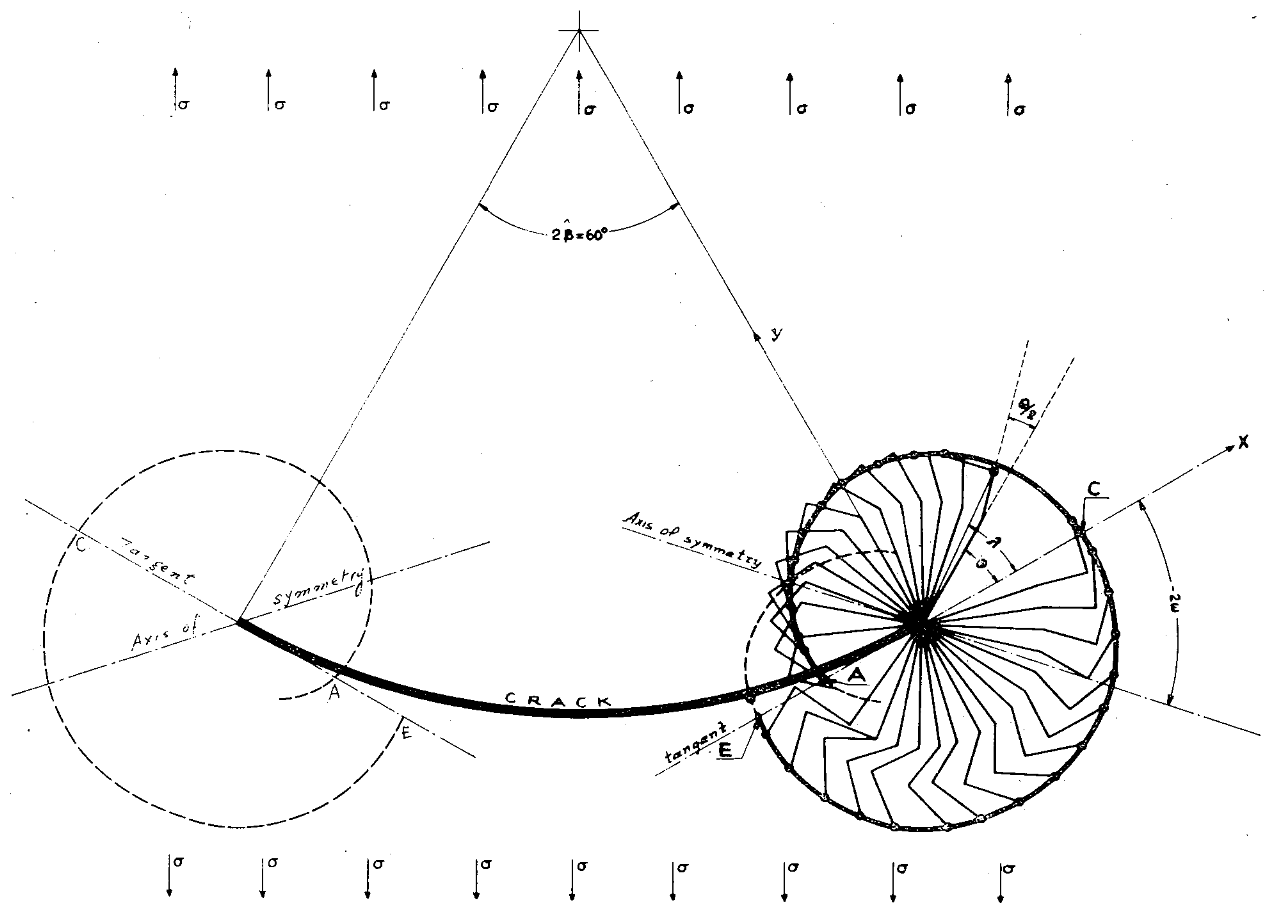

Fig 2. Crack chord perpendicular to σ. Geometrical construction of the caustic curve for the case of a 60° circular arc crack. Compare the inverse formation of the caustic in the case of an arc crack the chord of which is *parallel* to the direction of loading (cf. Fig 7(i) and 7(ii); Fig 8 and Fig 9).

Evaluation of K_i and K_{ii}

For each crack there exists a unique value of μ. As the loading increases each individual μ gives rise to a set of 'similar' caustics. Every member of the set have certain invariant geometrical properties, depending only on μ (not on the applied loading). It has been shown[9, 10] that the most sensitive property is the ratio AE/CE (Fig 2), where A, C, E, are the intercepts with the x axis. In the case of a circular arc crack, the axis coincides with the tangent at the crack tip; thus AE/EC is the ratio of the distances measured along the tangent (Fig 2). Rearranging equation (5),

$$\tan\lambda = \frac{3\sin\Theta + 2\sin\left(\frac{3\Theta}{2}+\omega\right)}{3\cos\Theta + 2\cos\left(\frac{3\Theta}{2}+\omega\right)}$$

(angles λ, ω, θ, are shown in Fig 2). Letting λ be 0, π, 2π, 3π, θ can be found since $\mu=\tan\omega$ is a given parameter in each case. Substituting these values in (6) the co-ordinates of A, C, E (and hence EC and EA) can be found with respect to r_o. Graphs can therefore be drawn showing the variation of $EC/r_o=\delta$ and (EC−CA)/EC versus μ[9] (Figs 3, 4). Thus it is possible to make use of these graphs to deduce the value of μ from the shape of the epicycloid by measuring the lengths EC and CA. The ratio $\frac{EC-CA}{EC}$ can be obtained from the photographs (Figs 7-10) μ is obtained from the corresponding graph and can thus be introduced to the second graph to determine $\delta=EC/r_o$. Since EC has already been measured, the value of r_o is obtained as,

$$r_o=\left\{\frac{3\,|C|\,(K_i^2+K_{ii}^2)^{1/2}}{2\sqrt{2\pi}}\right\}^{2/5}$$

giving $$r_o=\left\{\frac{3\,|C|\,K_i}{2\sqrt{2\pi}}\right\}^{2/5}\cdot(1+\mu)^{1/5}$$

from which the value of K^+ can be found. Solving for K_i and K_{ii}, since $|C|=dtc$,

$$|K_i|=\frac{1\cdot671}{dtc}(r_o)^{5/2}\frac{1}{(1+\mu^2)^{1/2}}$$
$$\text{and}\quad |K_{ii}|=\frac{1\cdot671}{dtc}(r_o)^{5/2}\frac{\mu}{(1+\mu^2)^{1/2}} \qquad (8)$$

The experiment to determine K_i and K_{ii}

For the study of the stress field, and the evaluation of the stress intensity factor at the crack tip, a series of experiments was performed on thin plates subjected to tension.

All specimens were made of plane sheets of perspex, 2 mm thick, 110 mm wide, 190 mm long. Slits in the form of arcs of radius 15 mm were cut with a fine thread saw, so that the tangent at the middle of the arc was at an angle of either 90° or 0° to the longitudinal axis of the specimen, and centred at the intersection of the diagonals. Several cases were considered in which the arc took the values $2\beta=30°$, 60°, 120°, 180° and 192°.

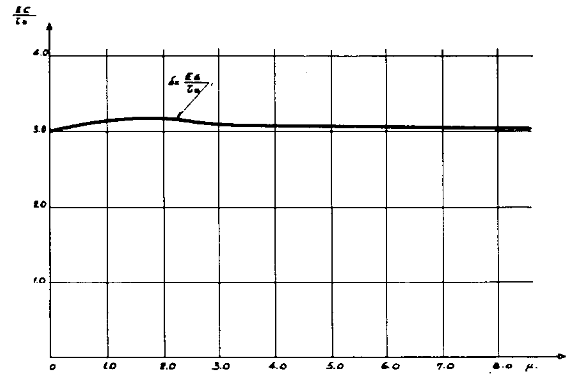

Fig 3. The variation of EC versus the ratio $-K_{ii}/K_i=\mu$. The above length is normalised to the radius r_o.

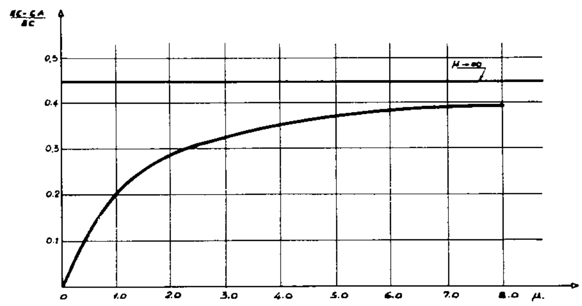

Fig 4. The variation of (EC−CA)/EC versus the ratio $-K_i/K_{ii}=\mu$.

The replacement of the stationary crack by a central slit is in agreement with the simple linear approach of the stress distribution considered here. The maximum slit (or chord) length is small as compared to the width of the specimen so that it may be assumed that the crack is remote from any boundary of the plate. In such an experiment the light used should possess certain properties: definite wavelength and intensity, such as the properties of a monochromatic and brilliant beam emitted from a L.A.S.E.R. source. The diameter of the light beam ought to be larger than the chord of the slit. Since the pencil of light coming directly out of the laser was very narrow and intense, it was necessary to eliminate light spots and convert the beam into the proper diameter and intensity. The final arrangement thus included the laser, a shutter, a diffuser, a pair of lenses, the specimens placed beyond the focal point, and a screen upon which photographic films were exposed to record the magnified sections of the caustics (Fig 6).

Since divergent light was used, the magnification ratio ρ had to be taken into account; ρ is equal to $(d+d_1)/d_1$, where d is the distance of specimen from screen and d_1 its distance from the focal point. Thus relations (8) become:

$$|K_i|=\frac{1\cdot671}{dtc}\,\frac{1}{\rho^{3/2}}\left(\frac{EC}{\delta}\right)^{5/2}\frac{1}{(1+\mu^2)^{1/2}}$$
$$|K_{ii}|=\frac{1\cdot671}{dtc}\,\frac{1}{\rho^{3/2}}\left(\frac{EC}{\delta}\right)^{5/2}\frac{\mu}{(1+\mu^2)^{1/2}} \qquad (9)$$

where $\frac{EC}{\delta}=r_o$

Results

Some expressions have been established[6] for the theoretical stress intensity in the case of an internal arc crack of radius R_c, 2βrad, subjected to tension. When the direction of the applied loading is *parallel* to the chord of the crack,

$$K_i=\frac{\sigma}{8}\sqrt{\pi R_c \sin \beta}\times \left\{\frac{1+\sin^2 \beta/2 \cos^2 \beta/2}{1+\sin^2 \beta/2}\cos \beta/2 - \cos\frac{3\beta}{2}\right\}$$

$$K_{ii}=\frac{\sigma}{8}\sqrt{\pi R_c \sin \beta}\times \left\{\frac{1+\sin^2 \beta/2 \cos^2 \beta/2}{1+\sin^2 \beta/2}\sin \beta/2 - \sin\frac{3\beta}{2}\right\} \quad (10)$$

When the direction of the loading is *perpendicular* to the chord of the crack, the respective expressions are:

$$K_i=\frac{\sigma}{8}\sqrt{\pi R_c \sin \beta}\times \left\{\frac{1-\sin^2 \beta/2 \cos^2 \beta/2}{1+\sin^2 \beta/2}\cos \beta/2 + \cos\frac{3\beta}{2}\right\}$$

$$K_{ii}=\frac{\sigma}{8}\sqrt{\pi R_c \sin \beta}\times \left\{\frac{1-\sin^2 \beta/2 \cos^2 \beta/2}{1+\sin^2 \beta/2}\sin \beta/2 + \sin\frac{3\beta}{2}\right\} \quad (11)$$

where σ is the stress at infinity.

Repeated experiments, tests, and measurements have corroborated the theory, there being agreement between the theoretical and the experimental data, as shown by the table below. The indicative values of both theoretical and experimental estimates of μ, $|K_i|/\sigma$, $|K_{ii}|/\sigma$, have been evaluated for the case of cracks of $\beta=15°$, $30°$, $45°$ and $90°$; in all the cases the chord was perpendicular to the direction of load. The tabulated values were produced from measurements on the photographs (Figs 7, 8, 9).

Discussion of results

The agreement between the theory and the experiments is satisfactory, indicating the accuracy of the method. Caustics corresponding to different values of σ were taken in order to increase the accuracy of the measured values of μ, which were obtained from the curves. K_i and K_{ii} are directly proportional to the applied stress, equations (10) and (11), but their ratio is not. Therefore by increasing σ similar curves

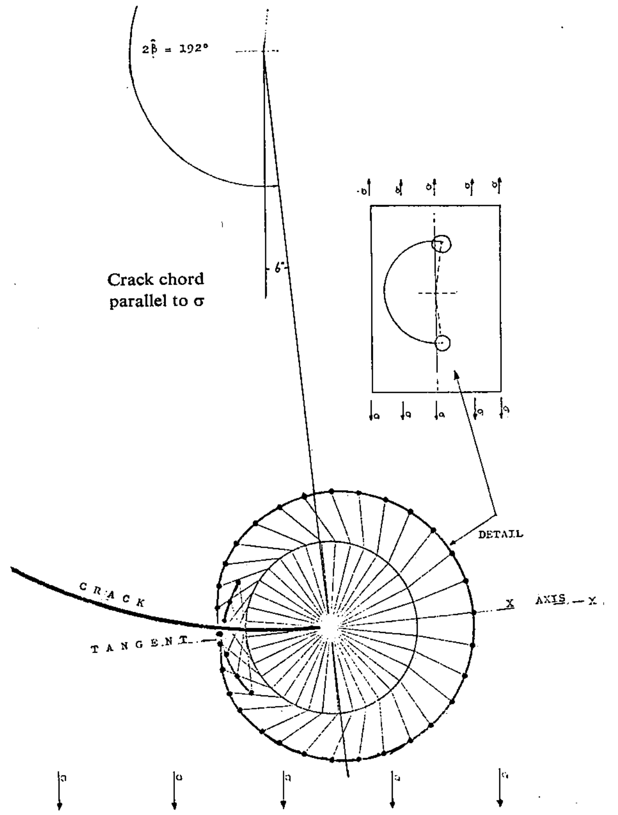

Fig 5. Chord parallel to σ; in the case of arc 192° the axis of symmetry of the caustic coincides with the tangent to the crack tip, thus zeroing K_{ii} (cf. Fig 10).

were obtained, having the same ratio AE/EC=a which remained fixed although the dimensions of the curves increased after each loading step. (See Fig 9 where a, b, ... onwards are successive loading steps.)

According to (9) AE/EC=a is a function of μ only and does not change; EC increases only when K_i and K_{ii} increase with σ. The photographs taken for various loading steps show the dimensional increase of the caustics (Figs 9(a), 9(b), 9(c), 9(d)).

In the case of cracks having their chords parallel to the direction of the loading, it has been established by expressions (10) and (11) (giving the theoretical estimates of K_i and K_{ii}), that for $\beta=96°$ one must expect $K_{ii}=0$ and $K_i\neq 0$, where $K_{ii}/K_i=\mu=0$. To test the above theoretical prediction a two times 96° arc was tested. As expected, the caustic curve obtained had an axis of symmetry rotated at an angle of $-2\tan^{-1}(\mu)=-2\tan^{-1}(0)=0°$ with respect to the x axis which, in this case, coincides with the

β	Theory	Experiment	Theory	Experiment	Theory	Experiment
deg. of arc	μ		$(\|K_i\|/\sigma)\times 10^2\ m^{-1/2}$		$(\|K_{ii}\|/\sigma)\times 10^2\ m^{-1/2}$	
15	0·5	0·53	2·51	2·49	1·26	1·3
30	1·2	1·1	2·97	2·64	3·56	2·9
45	2·0	2·1	1·86	1·7	3·7	3·6
90	3·0	3·0	0·96	0·95	2·98	2·85

tangent at the crack tip. Therefore the caustic curve looped on the tangent, as expected (Figs 5 and 10).

In the cases considered in this paper the curvature is kept constant. The value of the stress intensity factors, apart from the applied loading, depend on two additional factors:

first, the length of the arc (angle and radius of curvature)

second, the angle of inclination of the tangent at the crack tip, relative to the direction of the applied loading.

The dependence on the second factor only (angle of inclination) has been demonstrated by earlier experimenters using the method of caustics on straight line cracks (arc crack radius $\rightarrow \infty$). K_i and K_{ii} vary in response to the change of orientation of the chord with respect to the direction of loading.[9] In these cases the direction of the tangent at the crack tip coincides with the crack itself and its chord. When an arc slit of zero curvature (straight line crack), parallel to the direction of load, is constructed, no caustic will appear and both K_i and K_{ii} will be zero. On the other hand caustics will appear when a specimen of finite curvature $2\beta=180°$ is tested, the tangent at the crack tip being parallel to the direction of the loading, as in the straight line case. Here it is the first factor (curvature contribution) that gives rise to caustics which are smaller than the ones obtained by lesser arcs under the same loading conditions (see Fig 8 for a comparison of the size of caustics at loading step b; the load is the same; the area of the caustic formed by arc 60° is larger than the one formed by arc 90°).

The second factor (inclination of tangent) does not contribute to the value of the K's in the case of a parallel tangent at the tip of an arc of 180°.

Using a similar qualitative argument it can be shown why K_{ii} is not zero in the case of an 180° arc crack, where the tangent at the tip is perpendicular to the direction of loading. In the case of a straight line crack perpendicular to the applied loading, K_{ii} is equal to zero. For a $2\beta=180°$ arc crack, whose tangent at the tip is perpendicular to the direction of loading, K_{ii} is not equal to zero because of the contribution of the curvature to its value. The contribution of the tangent inclination is zero, as indicated by the zero curvature experiment. Now if β is increased to 96° (arc 192°), K_{ii} becomes zero, since the influence of the curvature is cancelled by the contribution of the added 6° inclination of the tangent (Figs 5 and 10 where arc is 192°).

Optical system and photographs

The optical system shown in Fig 6, is very simple. A source of intense monochromatic light (in this case a laser) emits a pencil of light through a shutter to a diffuser and then to a pair of lenses. The convergent light beam impinges, at incidence as near the normal as possible, on the specimen which is placed beyond the focal point. The reflected diverging beam forms an image of the crack on the screen (Figs 1 and 6). The singularity of the crack tip causes the formation of the caustic surface which is shown as a very bright cycloidic and epicycloidic curve surrounding the tip. The fringe patterns (see earlier section on 'The method of caustics' final paragraph) are not taken into consideration in this paper.

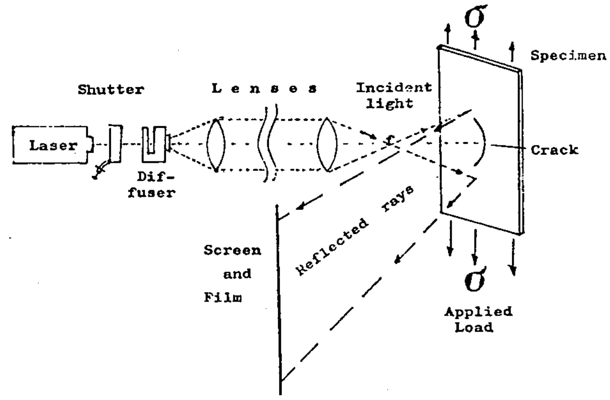

Fig 6. A schematic view of the optical system.

No camera is needed. Figs 7, 8, 9 and 10(i) are exposures of photosensitive films coating the screen surface. The direction of tension is always *perpendicular* to the long side of each photograph shown in this paper and letters a, b, c, . . . indicate progressive steps of loading. The variations of the area of the caustics and the ratio EA/AC with respect to angle β can be best understood if the photographs would be considered together with Figs 2, 3, 4 and 5.

To illustrate the results and discussion the photographs have been grouped according to the direction of the crack chord relative to the direction of loading. In the case of parallel direction, equation (10), the ratio EA/EC for $\beta=96°$ is zero, since EA=0 (Figs 5 and 10) and the sliding effect is not present. As the angle decreases the ratio EA/EC increases.

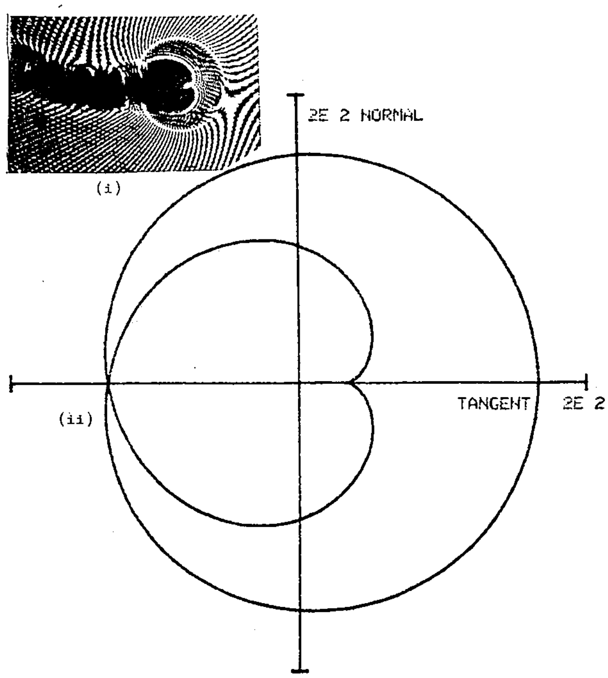

Fig 10. Caustics of an arc 192° the chord of which is *parallel* to the direction of load: (i) photographed; (ii) plotted by computer. In this case EA equals zero, consequently $\mu=0$ (cf. Fig 5).

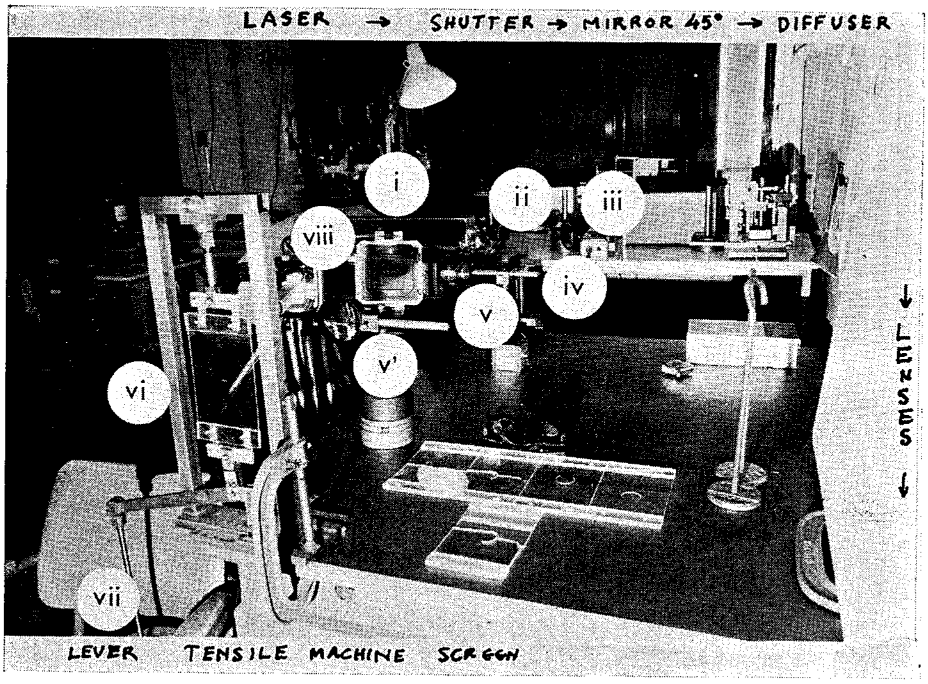

Fig 6a. The optical system: (i) laser; (ii) shutter; (iii) 45° plane mirror; (iv) diffuser; (v) lenses; (vi) tensile machine with specimen; (vii) tension lever; (viii) screen.

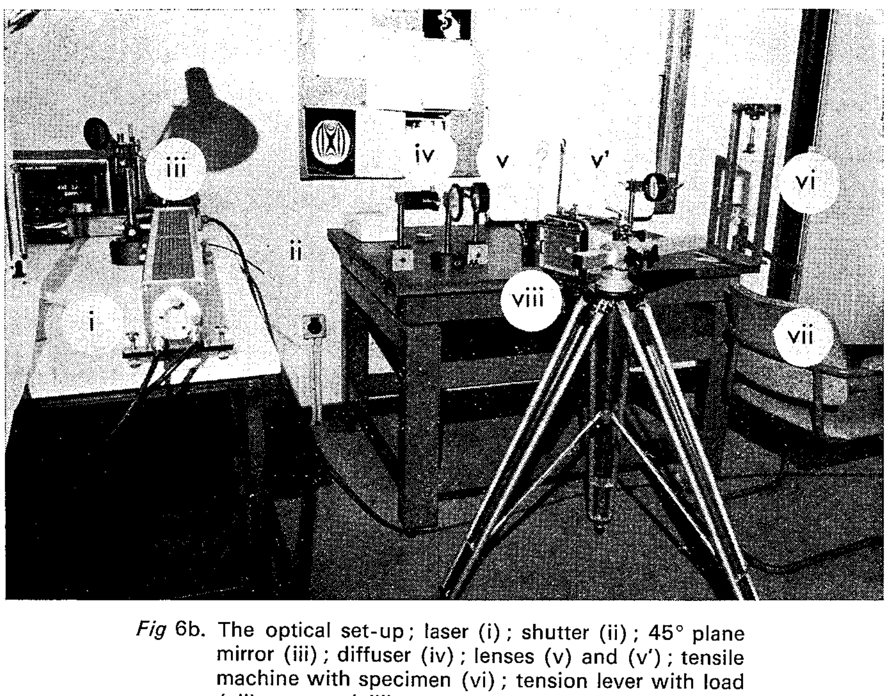

Fig 6b. The optical set-up; laser (i); shutter (ii); 45° plane mirror (iii); diffuser (iv); lenses (v) and (v'); tensile machine with specimen (vi); tension lever with load (vii); screen (viii).

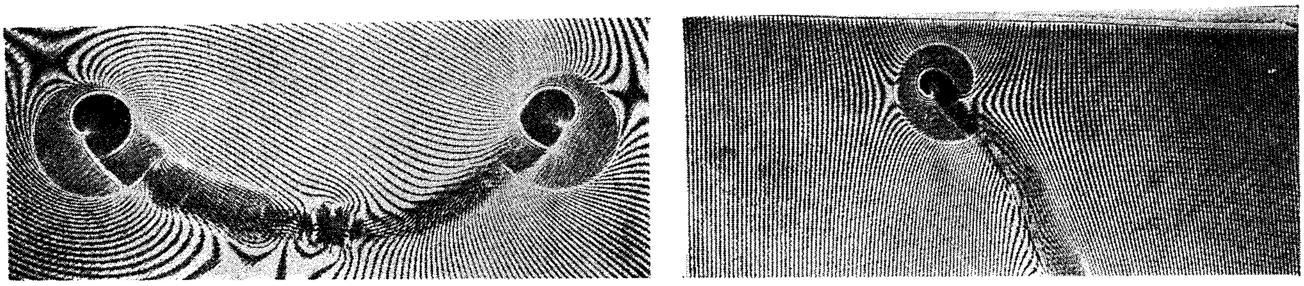

Fig 7. Inverse formation of caustics in arc cracks when: left (i) the chord is perpendicular and right (ii) the chord is parallel to the direction of loading.

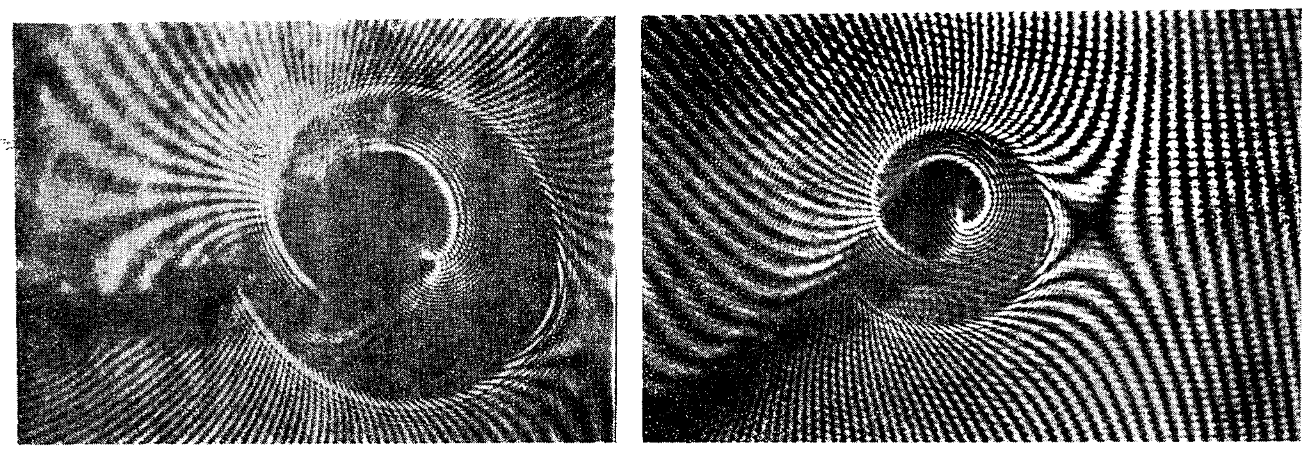

Fig 8. The effect of *angle* on the ratio EA/EC. Chord *perpendicular*. Left (i) crack-arc 60°; right (ii) crack-arc 90°.

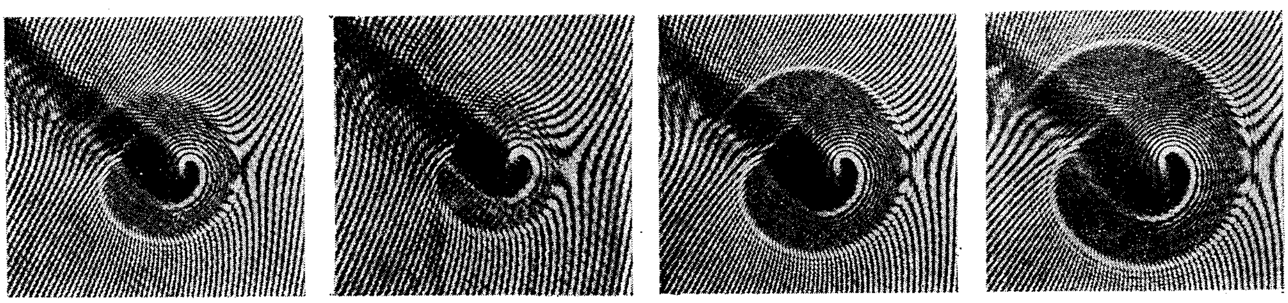

Fig 9. The effect of *load* on the size of caustic. Chord *parallel*, crack-arc 120°. Photographs of the same specimen show that the caustic increases (a, b, c, d) as the load increases, *but* the ratio EA/EC is the same.

When the directions are perpendicular both the angle and the ratio decrease. Thus in the case of 90° and 60° the increase of the sliding effect is conspicuous (Fig 8).

Another set of photographs (Figs 9 a, b, c, and d) show the increase in the area of the caustic as the loading increases although the direction, the angle and the ratio EA/EC remain the same.

Concluding remarks

1. The particular form of the crack tip under tension reflects a caustic onto a screen. The variations of the stress field cause infinitesimal elastic deformations of the tip surface which, in turn, yield various caustics. The experimental method, investigating the relation in the above, or the reverse order, proves that caustics, surface topography and stress field are causally connected.
2. The region of the elastic singularity at the crack tip, where steep stress gradients vary, is rather infinitesimal; therefore not amenable to accurate observation by means of the old methods. By the method of caustics the elastic singularity is transformed into an optical singularity; the latter permits the study of the stress field by means of the intensity factor ratio which is also important as a fracture- or crack propagation-criterion. Since K^+ (and $^{Ki}/_{Kii}$) is a function of the geometry of the crack and of the way the loading is applied, for certain values of these parameters K^+ reaches values showing whether the material under consideration will fail or not.
3. The experiments suggest a simple rule: no loading, no caustic; more loading, larger caustic.
4. The general theory for determining stress intensity factors in the case of straight cracks is also applicable to arc type cracks.
5. The value of the intensity factors as predicted by the theory, was found to depend on:
   (i) the curvature of the crack
   (ii) the subtended angle, and
   (iii) the inclination of the tangent relative to the direction of the applied loading.
6. The Theocaris' method of caustics permits accurate measurements. The difference between the theoretically predicted and the experimentally obtained results does not exceed $+0{\cdot}67\%$ for μ, $-5{\cdot}33\%$ for $|Ki|/\sigma$, and $-5{\cdot}6\%$ for $|Kii|/\sigma$, which could be attributed to usual measuring errors, to the laser beam not being normal to the specimen surface and to imperfections in the photographic film.
7. The technique used in this paper could be improved and extended to cover even more complicated geometry of cracks and loads in various materials such as metals. Crack propagation in shells and plates might be studied by the application of a similar procedure using high-speed cinematography to record the rapid change of caustics along the crack-propagation path.
8. The method appears to be versatile and applicable not only to elastic plane-stress problems but also to elastic-plastic cases and even to fully plastic problems.[14]

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