Optical Measurement of the Plastic Strain Concentration at a Crack Tip in a Ductile Steel Plate

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The shadow spot method has been established as a valuable experimental procedure for measuring elastic stress intensity factors in planar fracture specimens. It is noted here that if the crack-tip deformation field in specimens of ductile materials can be characterized by means of a single plastic intensity factor, analogous to the stress intensity factor in linear elastic fracture mechanics, then the shadow spot method has potential for use in measuring this intensity factor. The value of the J-integral is adopted as the plastic strain intensity factor, and the lateral contraction of an elastic-ideally plastic planar specimen is calculated in terms of J from the nonhardening limit of the HRR asymptotic field of elastic-plastic fracture mechanics. The theoretical caustic curve which would be generated by geometrical reflection of normally incident parallel light from points of the deformed specimen surface lying well within the plastic zone is determined, and it is shown that the value of J is proportional to the maximum transverse diameter of the shadow spot to the third power. Results of preliminary experiments, in which values of J for a given single edge notched steel plate specimen are inferred from measurements peformed separately from the elastically and plastically deforming parts of the specimen, are also reported.

1 Introduction

The use of the optical shadow spot method, which is also known as the method of caustics, for measuring the intensity of crack tip stress fields is now quite common in experimental work underlying the field of linear elastic fracture mechanics. The conceptual basis for use of the method in reflection will be reviewed in a later section, but the main features for linear elastic material response may be briefly stated in the following way. When a large plate containing a long through-crack is loaded so that crack opening occurs in mode I, the stress and deformation fields very near the crack tip assume the familiar universal spatial distribution. Only the magnitude of the near tip field varies with load and geometry, and this magnitude is customarily the mode I elastic stress intensity factor. Within the framework of plane stress analysis, the deformed shape of the specimen surface near the crack tip is thus also known up to a scalar amplitude which is equivalent to the stress intensity factor. The success of the method of caustics is based on the fact that, with a suitable optical arrangement, the light pattern obtained by reflecting parallel incident light from the specimen surface near the crack tip provides a direct measure of the stress intensity factor.

From the foregoing discussion, it is clear that applicability of the method does not hinge on the material in the crack tip region responding in a linear elastic manner. Instead, the key feature is that the deformed shape of the specimen surface (i.e., the reflecting surface) in the crack tip region is known up to a scalar amplitude. Asymptotic elastic-plastic analyses of near-crack-tip fields in power-hardening and nonhardening materials suggest that this situation may prevail for these cases as well. Within the framework of plane stress analysis, with small strains and proportional stress histories for stationary cracks, the value of Rice's J-integral has been proposed as a plastic intensity factor. The viewpoint is adopted here that J provides a suitable scalar amplitude for the deformed shape of the surface of an elastic-plastic fracture specimen, and a means of measuring this amplitude is proposed.

In the following sections, the results of asymptotic elastic-plastic crack tip analyses are summarized. The particular case of a nonhardening material in a state of plane stress is examined in detail, and the shape of the caustic curve obtained by reflection of parallel incident light from the crack tip region of the specimen surface is predicted. The results are compared to caustics obtained in the laboratory on specimens of a ductile steel which was chosen because its stress-strain response is described very well by an elastic-perfectly plastic material model. Finally, values of J inferred from measurements on a single specimen subjected to fixed loads with the initial curve far outside of the crack tip plastic zone and with the initial curve deep inside the plastic zone are compared. If the integral expression for J is indeed path-independent then the values inferred from these separate measurements should be identical, and they are found to be approximately equal.

2 Formation of Caustics in Reflection

Consider a family of light rays parallel to the x_3 axis in-

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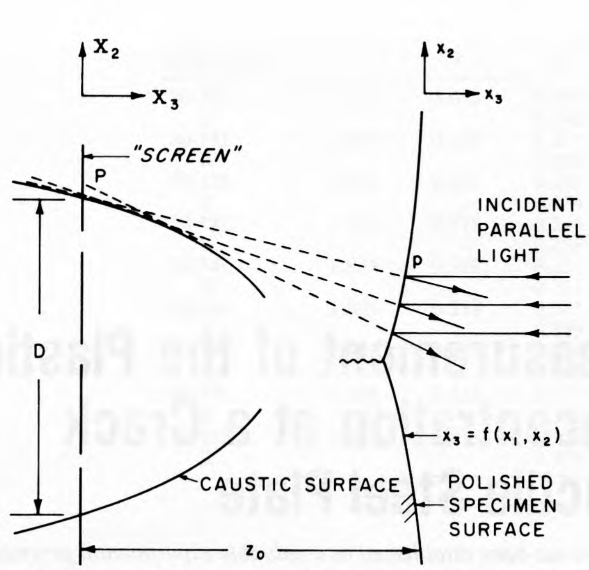

Fig. 1 Schematic of the optical setup for caustics by reflection

cident on the reflective surface $x_3 = f(x_1, x_2)$ of an opaque material; see Fig. 1. Upon reflection from the surface, the light rays will deviate from parallelism in general. If certain geometrical conditions are met by the reflecting surface, then the family of reflected rays (or their virtual extensions through the reflecting surface) will have an envelope in the form of a three-dimensional surface in space. This surface, which is called the *caustic surface*, is the locus of points of maximum luminosity in the reflected field or its virtual extension. The reflected rays are tangent to the caustic surface. If a "screen" is positioned parallel to the x_1, x_2 plane and so that it intersects the caustic surface, then a cross-section of the caustic surface can be observed as a bright curve (the so-called *caustic curve*) bordering a relatively dark region (the *shadow spot*) on the "screen." In practice, of course, there would be no image produced on a screen placed in the shadow of an opaque specimen. Instead, the plane of the "screen" would be the focal plane of a camera used to observe the reflected light field.

Suppose that the incident ray which is reflected from the point $p(x_1, x_2)$ on the reflecting surface intersects the screen at the image point $P(X_1, X_2)$; see Fig. 1. The X_1, X_2 coordinate system is identical to the x_1, x_2 coordinate system, except that the origin of the former has been translated the distance z_0 to the screen. The position of the image point P will depend on the slope of the reflecting surface at p and on the normal distance z_0, and it is given by

$$X_i = x_i - 2z_0(\partial f/\partial x_i) \tag{2.1}$$

where $z_0 >> |f|$. Equation (2.1) is a mapping of the points on the reflecting surface onto the points on the screen. If the screen intersects a caustic surface in the reflected light field, then the resulting caustic curve on the screen is a locus of points of multiple reflection. That is, for those points on the caustic curve, the mapping (2.1) is not invertible and the determinant of the Jacobian matrix of the transformation must vanish, i.e.,

$$\frac{\partial(X_1,X_2)}{\partial(x_1,x_2)} = 0 \tag{2.2}$$

The vanishing of the determinant of the Jacobian matrix is the necessary and sufficient condition for the existence of a caustic curve. The points on the reflecting surface for which the Jacobian vanishes are the points from which the rays forming the caustic curve are reflected. The locus of these points on the reflecting surface is the so-called *initial curve*. The method is reviewed more completely in a recent article by Kalthoff and Beinert [1].

3 Asymptotic Elastic-Plastic Crack Tip Field

A major advance in the investigation of the plastic fields which prevail in the vicinity of a crack tip in a ductile material was the introduction of the HRR singularity by Hutchinson [2] and Rice and Rosengren [3]. They considered the case of a monotonically loaded stationary crack in a material described by a J_2-deformation theory of plasticity and a power hardening relationship between plastic strain ϵ_{ij}^p and stress σ_{ij} of the form

$$\frac{\epsilon_{ij}^p}{\epsilon_0} = \frac{3}{2}\alpha\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0} \tag{3.1}$$

where

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\,\sigma_{ij}\ ,\ \sigma_e^2 = \frac{3}{2}s_{ij}\,s_{ij} \tag{3.2}$$

and σ_0 is the tensile yield stress, ϵ_0 is the equivalent tensile yield strain, n is the hardening exponent, and α is a material constant. By introducing the above assumptions, they observed that, within a small strain formulation, a possible asymptotic solution of the elastic-plastic field equations has the form

$$\begin{aligned} \epsilon_{ij} &\to \frac{\alpha\sigma_0}{E}\left[\frac{JE}{\alpha\sigma_0^2 I_n r}\right]^{\frac{n}{n+1}} E_{ij}(n,\theta) \\ \sigma_{ij} &\to \sigma_0\left[\frac{JE}{\alpha\sigma_0^2 I_n r}\right]^{\frac{1}{n+1}} \Sigma_{ij}(n,\theta) \end{aligned} \tag{3.3}$$

as $r \to 0$. The angular factors Σ_{ij} and E_{ij} in (3.3) depend on the mode of loading and on the hardening exponent. The dimensionless quantity I_n, which is defined in [2], decreases monotonically from 5 for $n = 1$ to 2.57 for $n \to \infty$ for cases of plane stress. J in (3.3) is the value of Rice's J-integral. The singular field given by (3.3) is customarily referred to as the HRR singularity.

For plane deformation, the J-integral is defined for any path of integration Γ by [4]

$$J = \int_\Gamma [Wn_1 - n_i\,\sigma_{ij}\,u_{j,1}]\,d\Gamma \tag{3.4}$$

where W is the local stress work density, n_i is a unit vector normal to Γ, and u_i is the particle displacement vector. The integral (3.4) has the well-known property of path-independence, that is, $J = 0$ for any simple closed path in the body in the absence of body forces. This implies that J has the same value for all paths which begin on one traction free face of a crack in a plane with normal in the x_2 direction and which end on the opposite traction free face of the crack. Because the path of integration can be chosen to be arbitrarily close to the crack tip, J has been interpreted as a measure of the strength of the crack tip singular field, a role which is obvious from the form of (3.3). Based on the observation that J is a characterizing parameter for the crack tip field, it has been suggested that a condition for onset of crack growth is the attainment of a critical value of J. This seems reasonable, provided that the one-parameter characterization remains valid and that the one-parameter field prevails over a region large in size compared to the fracture process zone size [5]. The interest in measuring values of J for ductile materials stems from the potential usefulness of this suggested fracture initiation criterion.

The asymptotic result (3.3) was derived under the further assumption that the dependence of the local field on the polar coordinates r and θ is indeed separable. For the case of antiplane shear, Rice [4] has shown that the asymptotic solution

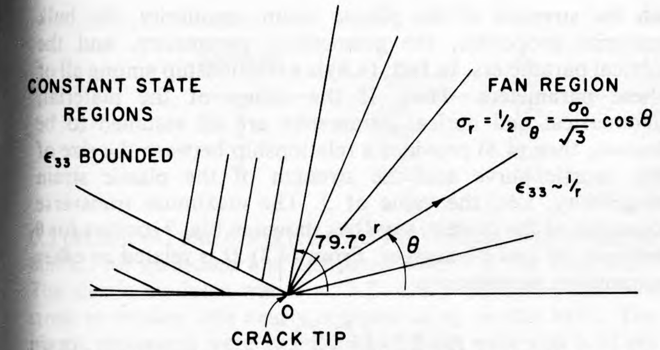

Fig. 2 Angular sectors of the asymptotic elastic-plastic solutions for $n \to \infty$ [6]

is separable. For the inplane cases, however, the equivalent result has not yet been established and (3.3) represents but one possible asymptotic solution. Results of numerical calculations for both incremental and deformation elastic-plastic theories indicate a behavior very close to the HRR singularity near the crack tip.

For the case of elastic-perfectly plastic materials Hutchinson [6] assembled candidate stress fields for plane stress and plane strain. The stress distributions that he chose were suggested by the behavior of the equivalent HRR fields in the limit $n \to \infty$ and were in accordance with the predictions of the rigid-plastic solutions available [6]. For plane stress, the chosen field consists of a centered fan sector extending from $\theta = 0$ to $\theta = 79.7$ degrees and two constant state regions filling the remaining sector from $\theta = 79.7$ degrees to $\theta = 180$ degrees; see Fig. 2. The individual sectors comprising the described assembly are of the same type as those proposed by Rice [7] as possible asymptotic solutions for a plane stress stationary crack in an incremental elastic-plastic material.

Here, the nonhardening limit of (3.3) with $n \to \infty$ will be considered, and the corresponding asymptotic field proposed by Hutchinson will be used as a basis for analysis. Evidence which is based on direct optical measurement within the crack tip plastic zone and which is consistent with the existence of the HRR crack tip singular field will be presented. Furthermore, an experimental procedure will be proposed for direct determination of the strength of the singularity, as represented by J, for elastic-perfectly plastic materials. It should be noted that the approach is valid for arbitrary values of the hardening exponent n. The analysis is greatly simplified if the special case $n \to \infty$ is considered, however, and attention is restricted to this special case for the time being in order to examine the procedure in its simplest and most transparent form.

4 Caustic Curves Due to HRR Field

Most structural alloys which contain cracks undergo substantial plastic deformation under rising load prior to onset of crack growth. In many cases, including standard specimen configurations, a large plastic zone develops around the crack tip and there is no region of the body over which a stress intensity factor controlled elastic field prevails. The linear elastic fracture toughness approach to fracture resistance characterization is then not applicable and other criteria, such as the critical J criterion mentioned in the previous section, must be adopted. Methods for measuring values of J for ductile fracture specimens are available but the methods are indirect, in general, in the sense that values of J are inferred from other measured quantities, typically load-deflection data. In this section, attention is focused on points deep within the crack tip plastic zone, and a means of inferring values of J from the local deformation field is suggested. This capability would be of great advantage in dynamic fracture testing of ductile materials because, in the interpretation of such experiments, compliance and other methods based on a quasi-static idealization of the deformation are not generally applicable.

Consider a large plate of elastic-perfectly plastic material with uniform thickness d in the undeformed state. Suppose that the plate contains a long through-crack and that the plate is subjected to edge loading which produces a plane stress opening mode of deformation. If a cartesian coordinate system is introduced so that $x_3 = 0$ is the mid-plane of the plate, then the normal displacement $u_3(x_1, x_2)$ of the plate face initially at $x_3 = d/2$ is

$$u_3(x_1, x_2) = -f(x_1, x_2) = \frac{d}{2}\epsilon_{33} = \frac{d}{2}(\epsilon_{33}^e + \epsilon_{33}^p) \tag{4.1}$$

where ϵ_{33} is the total strain in the thickness direction. The elastic strain is bounded everywhere, so that in sectors near the crack tip where the plastic strain is singular, the elastic strain is negligible by comparison. For such regions, the total strain in (4.1) is replaced by the plastic strain in the thickness direction. The assumption of plastic incompressibility, which is already incorporated in (3.1), requires that

$$\epsilon_{33}^p = -(\epsilon_{rr}^p + \epsilon_{\theta\theta}^p) \tag{4.2}$$

If the expressions (3.3) are substituted into the power hardening constitutive relation (3.1), then the plastic strain components in polar coordinates are given in terms of the corresponding stress components by

$$\begin{aligned} \epsilon_{rr}^p &= \frac{\alpha\sigma_0}{E}\left[\frac{JE}{\alpha\sigma_0^2 I_n r}\right]^{\frac{n}{n+1}}\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\left(\Sigma_{rr} - \frac{1}{2}\Sigma_{\theta\theta}\right) \\ \epsilon_{\theta\theta}^p &= \frac{\alpha\sigma_0}{E}\left[\frac{JE}{\alpha\sigma_0^2 I_n r}\right]^{\frac{n}{n+1}}\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\left(\Sigma_{\theta\theta} - \frac{1}{2}\Sigma_{rr}\right) \end{aligned} \tag{4.3}$$

As n becomes indefinitely large, I_n approaches the value 2.57 and the effective stress σ_e approaches the tensile flow stress σ_0 in plastically deforming regions. However, the limit of $(\sigma_e/\sigma_0)^{n-1}$ is indeterminate and apparently no simple description of the plastic strain distribution in the nonhardening limit may be extracted. Thus, further progress must rely on the result of detailed numerical calculations of the plane stress elastic-plastic crack tip fields. Fortunately, numerically determined strain distributions are available for $n = 25$ [8] and these results are adopted as good approximations for a nonhardening material. The numerically calculated stress distributions for $n = 25$ differ only slightly from the analytical nonhardening limit which can be extracted easily [6].

From (4.1) and (4.2), it is clear that the shape of the reflecting surface is determined by $\epsilon_{rr}^p + \epsilon_{\theta\theta}^p$ in the crack tip region. For $n \to \infty$, each strain component varies with the inverse of radial distance from the crack tip, that is,

$$\epsilon_{33} = \frac{J}{\sigma_0 I_n}\left(\frac{g(\theta)}{r}\right) \tag{4.4}$$

Thus, $g(\theta)$ is the numerically determined angular variation of $-(\epsilon_{rr}^p + \epsilon_{\theta\theta}^p)$ as presented in [8].

Suppose now that parallel light is normally incident on the surface of the plate, and that the surface is a reflecting surface. Equation (2.1) provides the mapping of points from the reflecting surface onto points on a "screen" positioned a distance z_0 behind the reflecting surface, according to the principles of geometrical optics. For the problem at hand, this mapping takes the form

$$X_i = x_i + \frac{Jdz_0}{\sigma_0 2.57}\frac{\partial}{\partial x_i}\left(\frac{g(\theta)}{r}\right) \tag{4.5}$$

According to the near tip stress distribution proposed by

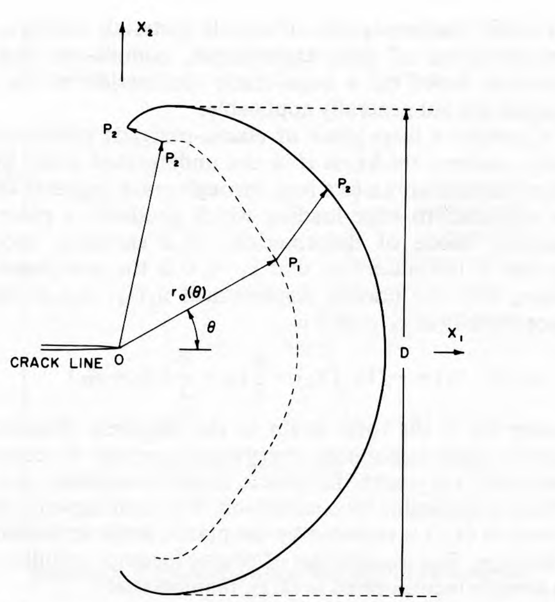

Fig. 3 Geometrical construction of the predicted initial curve (dashed) and caustic curve (solid).

Hutchinson [6] for an elastic-perfectly plastic material, the plastic strains are singular only in the sector $|\theta| < 79.7$ degrees. In other sectors, the strains are locally uniform and thus cannot contribute to the formation of the caustic surface. Within the singular sector, the first stress invariant is

$$\sigma_{rr} + \sigma_{\theta\theta} = \sigma_0 \sqrt{3} \cos \theta \tag{4.6}$$

If the differentiation indicated in (4.5) is carried out, and if the determinant of the Jacobian matrix of the transformation (4.5) is set equal to zero, the result is a quadratic equation for r^3 in which the coefficients depend on $g(\theta)$, $g'(\theta)$, and $g''(\theta)$, where the prime denotes derivative with respect to θ. The root of the quadratic equation which is of interest here is

$$r^3 = [r_0(\theta)]^3 = C\{ -(g'' + g) + \sqrt{(3g - g'')^2 + (4g')^2} \} \tag{4.7}$$

where $C = Jdz_0/5.14\sigma_0$. The radius of the critical curve on the specimen surface is $r_0(\theta)$. In terms of the initial curve radius, the mapping (4.5) is

$$\begin{aligned} X_1 &= r_0 \cos \theta + 2Cr_0^{-2}(g \cos \theta + g' \sin \theta) \\ X_2 &= r_0 \sin \theta + 2Cr_0^{-2}(g \sin \theta - g' \cos \theta) \end{aligned} \tag{4.8}$$

Both (4.7) and (4.8) hold for $|\theta| < 79.7°$.

The shape of the initial curve and the caustic curve were determined by evaluating numerically the expressions in (4.7) and (4.8). The function $g(\theta)$ is given in [8] (for $n = 25$) for increments in θ of 1 degree. The derivatives g' and g'' were determined from this data by means of standard central difference formulas. The results of the calculation are shown in Fig. 3, where the initial curve is shown as a dashed curve, and the corresponding caustic is shown as a solid curve. Analytically, equations (4.7) and (4.8) together describe the caustic curve formed on the screen by reflection of incident light from points on the specimen surface lying deep within the crack tip plastic zone, that is, inside the domain of validity of the HRR singularity. Obviously, (4.8) is a parametric representation of the caustic curve in the X_1, X_2-plane where the parameter θ varies over values in the range $|\theta| < 79.7°$.

The shape of the caustic curve depends only on the distribution of plastic strain in the crack tip region. The absolute size of the caustic curve, on the other hand, depends on the strength of the plastic strain singularity, the bulk material properties, the geometrical parameters, and the optical parameters. In fact, (4.8) is a relationship among all of these parameters. Thus, if the values of the material, geometrical and optical parameters are all assumed to be known, then (4.8) provides a relationship between the size of the caustic curve and the strength of the plastic strain singularity, i.e., the value of J. The maximum transverse diameter of the caustic, say D as shown in Fig. 3, occurs for θ between 56 and 57 degrees. From (4.8) D is related to other parameters according to

$$D = 2.38 \left(\frac{Jz_0 d}{\sigma_0} \right)^{1/3} \tag{4.9}$$

If D is adopted as the characteristic dimension of the caustic curve, then (4.9) may be solved for J to yield

$$J = \frac{\sigma_0 D^3}{13.5\, z_0 d} \tag{4.10}$$

Relation (4.10) is the main result of this section. It provides a simple relationship between the size of an observed caustic and the strength of the plastic strain concentration near the tip of a crack in an elastic-perfectly plastic material under plane stress conditions. For the result to be useful, it must be demonstrated that the values of J determined according to (4.10) and values determined according to other established procedures are the same. Some preliminary experimental evidence which suggests that this is indeed the case is presented in the next section.

5 Experimental Observations

Experiments of a preliminary nature were conducted in order to determine whether or not the direct measurement of values of J based on the foregoing analysis is feasible. The experiments were performed on single edge notched steel plate specimens in the double cantilever beam configuration which were loaded in a standard wedge loading arrangement. It was important for this investigation that the behavior of the material used was adequately described by an elastic-perfectly plastic model. A tool steel was chosen with a chemical composition of 0.9 percent C, 1.12 percent Mn, 0.3 percent Si,

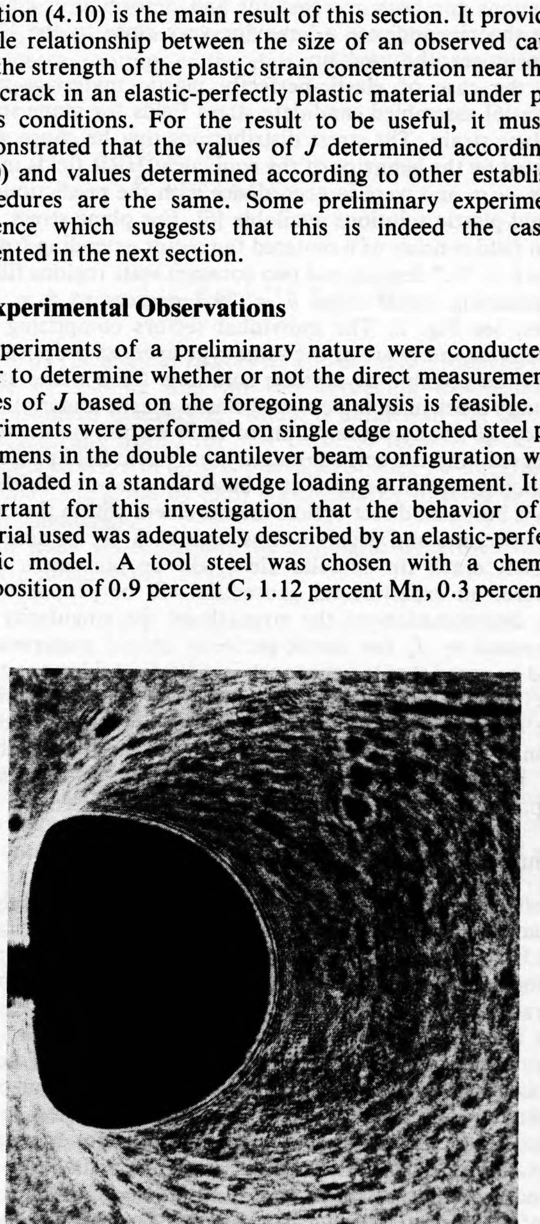

Fig. 4 Photograph of shadow spot, surrounded by slip lines, obtained by reflection from points well within the plastic zone

Table 1

Specimen	r_p cm	r_0 cm	D cm	z_0 cm	λ	J kPa•m
2	.102	.148	.468	41.4	1.41	12.9 (elastic)
2	.102	.040	.104	.127	12.65	13.3 (plastic)
3	.0795	.123	.388	33.54	1.9	7.68 (elastic)
3	.0795	.031	.081	.11	16.0	6.93 (plastic)

0.5 percent W, and 0.3 percent V. The material was heated to 800°C, oil quenched and tempered to 650°C for one hour. The elastic modulus was $E = 2.2 \times 10^5$ MPa and the flow stress in tension was nearly constant at $\sigma_0 = 970$ MPa. The plastic specimens were 10 cm long, 6.2 cm wide and 0.48 cm thick. A sharp pre-crack was cut along the long symmetry axes of the specimens with a spark cutter, and one surface of each of the specimens was polished to a mirror-like finish. Observations were made at points on the specimen which were far from the crack tip compared to the root radius of the crack.

A typical shadow spot obtained by reflecting light from the crack tip region of a loaded specimen is shown in Fig. 4. The shadow spot is surrounded by slip lines which define the visible extent of the crack tip plastic zone, and a lower bound on the extent of the actual plastic zone. The boundary of the shadow spot is produced by light reflected from deep within the plastic zone, where the HRR near tip field description can be expected to be valid. The shape of the observed shadow spot and bounding caustic curve compares very well with the caustic curve shown in Fig. 3, which was predicted on the basis of the HRR singular field. In view of this favorable qualitative comparison, two specimens were tested in order to obtain quantitative estimates of J.

Of course, the distances observed in the experimental setup depend on the optical arrangement used to observe them. Distances are measured as they appear in the film plane of the camera used to photograph the setup, and these distances are related to absolute distance through an amplification factor. The amplification factor is defined for a given setup as the ratio of the distance between the reference points as they appear on a photograph of the specimen and the actual distance between the reference points on the specimen. Distances measured in the film plane are converted to absolute distances by dividing by the amplification factor λ.

Results for two specimens are shown in Table 1. In this table, the length r_p represents the observed extent of surface evidence of plastic deformation and is interpreted as a measure of the plastic zone size. The size of the initial curve is denoted by r_0 and its value is determined as 0.316D based on an elastic model and as 0.363D for $\theta = 0$ from (4.7) for the plastic model. The maximum transverse diameter of the shadow spot D was determined from a photograph, and the screen distance z_0 was measured for each arrangement of the lenses and camera. The amplification factor λ for each setup is also shown in the table. With reference to Table 1, it should be pointed out that D is not measured directly. Instead, the values of λ and λD are measured and the corresponding value of D is inferred. The value of λD is much larger than D, in general, and therefore can be measured with greater certainty. For those cases in which the initial curve was well within the plastic zone, a value of J was inferred from (4.10). On the other hand, for those cases in which the initial curve was well outside the plastic zone, a value of J was inferred from the corresponding formula for purely elastic material response, that is,

$$J = 8.72 \times 10^{-3} \frac{ED^5}{\nu^2 d^2 z_0^2} \tag{5.1}$$

where E is the elastic modulus and ν is Poisson's ratio. These preliminary results are discussed in the next section.

6 Discussion

The ratio r_0/r_p had the values 1.48 and 1.55 for the elastic measurements on specimens 2 and 3, respectively, while this ratio had the values 0.393 and 0.389 for the plastic measurements on the same specimens. Thus, for the plastic measurements, the initial curve was indeed deep within the crack tip plastic zone where the HRR field could be expected to provide an accurate description of the local deformation field (with the proviso that this is in fact the correct asymptotic near tip field for elastic-plastic deformation). For the elastic measurements, the distance from the crack tip to the initial curve directly ahead of the crack tip was roughly 50 percent greater than the distance from the tip to the edge of the observed plastic zone. It would have been preferable to have had the ratio r_0/r_p somewhat larger for these measurements, but that could not be achieved in these preliminary tests. It was shown in [9] that interpretation of caustics data on the basis of an elastic analysis could be expected to yield valid results for the stress intensity factor provided that the initial curve radius r_0 was more than about twice the plastic zone size r_p. The analysis in [9] was based on the Dugdale-Barenblatt strip plastic zone model. The error introduced through the neglect of plasticity effects in the interpretation of caustics did not become extremely large, however, until the ratio r_0/r_p was reduced to a value of about 1.3. Thus the results of elastic measurements reported here are believed to be approximately correct in this respect.

For a given specimen subjected to fixed loads, the value of the J-integral is the same for all paths which begin on one face of the crack and which end on the opposite face of the crack as long as the particle stress histories are proportional during the loading process. Detailed elastic-plastic numerical calculations [10] have shown that this is the case for all material particles whose distance from the crack is greater than the crack tip opening displacement. Within the context of the present discussion, if the J-integral is path-independent then the value of J which was inferred from an elastic measurement and the value which was inferred from a plastic measurement should be equal. It is emphasized that both the elastic measurement and the plastic measurement are made on a *single* specimen subjected to a *fixed* load. The *only* difference between the two measurements is the positioning of the lenses, which results in an initial curve outside the plastic zone for the elastic measurement but an initial curve inside the plastic zone for the plastic measurement. It is clear from the results shown in Table 1 that the two values are approximately the same. Thus there seems to be adequate basis for pursuing this matter further. The fact that the elastic value of J depends on the measured value of D to the fifth power implies that there is a high degree of uncertainty in these results. For example, an error of 5 percent in the measurement of D for the calculation of an elastic value of J translates into a 28 percent error in the inferred value of J. For this reason, it would seem desirable in pursuing this experimental technique further to use an independently measured plastic value of J for comparison to the results of the optical measurements, instead of using an elastic value of J. On the other hand, it is clear from (4.10) that the plastic value of J depends on the measured value of D to the third power. Thus inferred values of J are much less sensitive to errors in D than in the elastic case.

Two matters which arose in applying the experimental method should be pointed out. First, plastic deformation in the crack tip region caused the polished specimen surface to lose some of its reflectivity in that region. The quality of the surface was still adequate to yield a well-defined shadow spot, but the photographs were not as clear as those commonly obtained when using the method of caustics to measure elastic stress intensity factors in polished steel specimens. Second, the normal distance z_0 from the undeformed reflecting surface of the specimen to the plane of the "screen" was very small for the setup used, on the order of the magnitude of D.

The purpose of this work is to suggest the possibility of using the optical method of caustics for direct measurement of J in a crack tip plastic zone in a ductile material. Clearly, further work is required before the approach can be regarded as an established procedure. On the experimental side, further evidence that the values of J deduced by means of the caustics method are the same as those determined by means of other standard methods is needed. Perhaps the values of J deduced from overall load-deflection data [11] obtained during loading of yielding precracked specimens can be used for this purpose. In addition, theoretical studies should be pursued toward predicting the shadow spots which would be observed for any amount of strain hardening (within the framework of power law hardening, say) and for the inital curve at any position within or outside of the crack tip plastic zone. It is only through such investigations that the ideas proposed here can be established as a basis for an acceptable procedure.

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References

1 Beinert, J., and Kalthoff, J. F., "Experimental Determination of Dynamic Stress Intensity Factors by the Method of Shadow Patterns," for publication in *Mechanics of Fracture*, Vol. VII, Edited by G. C. Sih, Noordhoff International Publishing, Leyden, Netherlands.

2 Hutchinson, J. W., "Singular Behaviour at the End of a Tensile Crack," *Journal of the Mechanics and Physics of Solids*, Vol. 16, 1968, pp. 13-31.

3 Rice, J. R., and Rosengren, G. F., "Plane Strain Deformation Near a Crack Tip in a Power Low Hardening Materials," *Journal of the Mechanics and Physics of Solids*, Vol. 16, 1968, pp. 1-12.

4 Rice, J. R., "Mathematical Analysis in the Mechanics of Fracture," in *Fracture: An Advance Treatise*, Vol. 2, H. Liebowitz (ed.), Academic Press, 1968, pp. 191-311.

5 Rice, J. R., "Elastic-Plastic Fracture Mechanics," in *The Mechanics of Fracture*, edited by F. Erdogan, ASME AMD Vol. 19, 1976, ASME, pp. 23-53.

6 Hutchinson, J. W., "Plastic Stress and Strain Fields at a Crack Tip," *Journal of the Mechanics and Physics of Solids*, Vol. 16, 1968, pp. 337-347.

7 Rice, J. R., "Elastic-Plastic Crack Growth," Report MRL E-124, Division of Engineering, Brown University (1980); also to be published in *The Mechanics of Solids*, The Rodney Hill 60th Anniversary volume, edited by H. G. Hopkins and M. J. Sewell, Pergamon Press, Oxford; May 1981.

8 Shih, C. F., "Elastic-Plastic Analysis of Combined Mode Crack Problems," Ph.D. thesis, Harvard University, Oct. 1973.

9 Rosakis, A. J., and Freund, L. B., "The Effect of Crack Tip Plasticity on the Determination of Dynamic Stress Intensity Factors by the Optical Method of Caustics," ASME *Journal of Applied Mechanics*, Vol. 48, 1981, pp. 302-308.

10 Parks, D. M., "Some Problems in Elastic-Plastic Finite Element Analysis of Cracks," Ph.D. thesis, Brown University, June 1975.

11 Rice, J. R., Paris, P. C., and Merkle, J. G., "Some Further Results of J-Integral Fracture Toughness Testing," ASTM STP 536, American Society for Testing and Materials, 1973, pp. 231-245.