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On radiation-free transonic motion of cracks and dislocations

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Abstract

Eshelby has shown that a glide dislocation can move without radiation of energy at $\sqrt{2}$ of the shear wave speed. It is also known that the same velocity plays a special role in shear crack propagation. This result has not received wide attention in the past due to lack of experiments and numerical simulations of transonic defects. Recent experiments on transonic shear fracture and molecular dynamics simulations of dislocation motion have stimulated renewed interest in the behavior of cracks and dislocations beyond the subsonic regime. We attempt to provide a unified treatment of transonic cracks and dislocations by elaborating on the fundamental result of Eshelby. We develop a unified treatment of radiation-free transonic motion of both cracks and dislocations. We use Stroh's method to generalize the Eshelby theorem to orthotropic and anisotropic elastic solids. In the case of orthotropic solids, we provide a proof of existence of the radiation-free speed. In the case of general anisotropic solids, there are three wave speeds $c_3 < c_2 < c_1$ in any given crystal orientation at which a moving defect is considered. In the first transonic regime $c_3 < v < c_2$, we show that there always exists a radiation-free state for any given velocity v of a moving defect. In the second transonic regime $c_2 < v < c_1$, the existence of radiation-free states appears to depend on both the symmetry properties of the material and the defect orientation. Examples of existence in the second transonic regime include a crack

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propagating in an isotropic solid and a crack propagating along a plane of symmetry in an orthotropic solid.

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1. Introduction

The mechanics of dynamic fracture has been of significant interest to the understanding of dynamic failure of solids and earthquakes. Although subsonic crack propagation has been thoroughly investigated by analytical, numerical and experimental methods, there is relatively little study on transonic (intersonic) and supersonic fracture (i.e., the crack tip velocity faster than the shear wave speed of the solid). Analysis of seismic data taken during crustal earthquakes shows that shear fracture on a pre-existing fault (weak plane) can propagate a velocity larger than the shear wave speed of the crustal material (e.g., Archuleta, 1982). This has led several researchers to investigate transonic crack growth in elastic homogeneous and isotropic solids (e.g., Burridge, 1973; Burridge et al., 1979; Freund, 1979, 1990; Slepyan, 1981; Simonov, 1983; Georgiadis, 1986; Broberg, 1989). It is established that there are two shock waves propagating with the crack tip when the crack tip velocity exceeds the shear wave speed. Unlike subsonic crack growth, stresses are singular not only at the crack tip, but also on shock waves, which represent lines of strong discontinuity. Another unique feature of transonic crack growth is that the crack tip singularity is not the square-root singularity as in quasi-static or subsonic dynamic crack growth, but depends on the crack tip velocity. In Mode I, both the power of the stress singularity and the dynamic crack tip energy release rate vanish as soon as the crack velocity exceeds the Rayleigh wave speed. In Mode II, the power of the stress singularity is always less than 1/2 except at a single crack tip velocity of $\sqrt{2}c_s$, at which the classical square-root singularity is preserved, where c_s is the shear wave speed of the material. However, there have not been any experiments that can validate the attainment of this crack speed in elastic homogeneous and isotropic solids because a crack always kinks or branches out, deviating from the initial crack plane and having a zig-zag crack path, once the crack tip velocity exceeds only 0.3–0.4c_s (e.g., Freund, 1990; Gao, 1993). Therefore, a wavy crack instability or crack branching always steps in before the crack tip velocity exceeds, c_s. This has prevented laboratory experiments from detecting transonic crack growth in elastic homogeneous and isotropic solids. The only possibility of attaining transonic conditions is to introduce a weak path (a layer of lower toughness) such that crack growth is confined to this weak path so that crack branching is prevented even in elastic isotropic materials.

To check this hypothesis, Rosakis et al. (1998) performed dynamic high-speed photoelastic experiments on weakly bonded, identical Homalite-100 plates loaded in an asymmetric impact configuration. They were able to show that under certain combinations of impact velocity and bond strength, shear dominated transonic cracks are generated and grow along the weak interface. The crack growth velocities were found to vary transiently and to cover the entire transonic regime attaining a maximum just short of the longitudinal wave speed c_d of Homalite-100 (~2100 m/s). The cracks were again accompanied by well-formed shear shock waves emanating from the crack tip, characteristic of all intersonically growing disturbances. This study clearly demonstrated that Mode-II crack growth is possible even in bonded, identical isotropic solids as long as weak crack paths are available, which are there to allow for Mode-II crack growth and to prevent crack branching.

Recently, Coker and Rosakis (1998) and Liu et al. (1998) have studied the various aspects of dynamic failure of unidirectional fiber-reinforced Graphite/Epoxy composite materials. On the macroscopic scale, which is much larger than the fiber diameter and fiber spacing, the composite can be considered as an elastic orthotropic solid. Its shear wave speed is 1920 m/s, while the longitudinal wave speeds normal and parallel to the fiber direction are 2800 m/s and 10,000 m/s, respectively, under plane-stress deformation. It is observed that, even under macroscopic mixed mode loading, a crack always propagated along the fiber direction. This is because the fiber/matrix interfaces are much weaker than the fibers in these composites such that the interfaces form weak crack paths for crack propagation. In Mode-I dynamic fracture, the crack tip velocity never exceeded the Rayleigh wave speed, i.e., transonic crack growth never occurred in Mode I, regardless of the effort to increase the impact velocities of the projectile. Furthermore, the measured dynamic crack tip energy release rate was found to decrease monotonically with the crack tip velocity up to the Rayleigh wave speed. In Mode II, however, the crack tip velocity not only exceeded the shear wave speed of 1920 m/s, but also transiently approached a much higher speed of about 9000 m/s and finally settled to a steady state speed of about 8000 m/s and remained at this speed for a substantial period of time in the experiment.

Motivated by these experimental studies, Huang et al. (1999) investigated the asymptotic field near a crack tip propagating intersonically in a homogeneous, elastic orthotropic solid. The crack tip velocity is between the shear wave speed and the higher longitudinal wave in the stiff (fiber) direction of the orthotropic solid. In Mode II, the asymptotic analysis shows that there exists a single crack tip velocity (higher than the shear wave speed) that gives a finite, non-vanishing crack tip energy release rate. At all other intersonic crack tip velocities the required crack tip energy release rate is zero. An expression of this critical velocity is given analytically in terms of the mechanical properties of the orthotropic composite. For the Graphite/Epoxy unidirectional fiber-reinforced composites, the predicted critical crack tip velocity agrees well with the stable crack tip velocity at which the crack grew in a near steady state for a substantial period of time in shear load dominated experiments (Coker and Rosakis, 1998). Therefore, based on the requirement of a finite, non-vanishing crack tip energy release rate, Huang et al. (1999) concluded that Mode-I intersonic crack propagation is impossible, and Mode-II intersonic crack growth tends to approach a stable crack tip velocity which depends only on the material properties.

From the point of view that a steady state crack can be modeled as a continuous array of Somigliana dislocations moving at the same speed, it is curious to see if the behavior of a transonic crack can be linked to that of a transonic dislocation. Eshelby (1949) has shown that there exists a critical velocity equal to $\sqrt{2}c_s$ at which the stress field around a uniformly moving glide dislocation is of purely subsonic form such that the dislocation moves without radiation of energy. Weertman (1967, 1969) studied transonic motion of both glide and climb dislocations and found that the Eshelby speed does not apply to a moving climb dislocation. Payton (1989, 1995) studied transient and steady state stresses around a transonic glide dislocation moving in a transversely isotropic elastic solid and showed that the radiation disappears at a critical speed. Molecular dynamics studies have shown that dislocations often propagate beyond the subsonic regime. Hoover et al. (1977) applied non-equilibrium molecular dynamics to the generation and steady propagation of edge dislocations and reported both subsonic and transonic dislocation propagation. Zhang et al. (1995) reported dislocation emission from a crack tip at both subsonic and transonic speeds. Recent molecular dynamics simulations by Gumbsch and Gao (1999) have shown that dislocations can move transonically and even supersonically (faster than both shear and longitudinal wave speeds) if they are subjected to sufficiently high shear stresses and if they are created as transonic dislocations by strong stress concentration. In the case of transonic motion, Gumbsch and Gao observed dislocation velocities around $\sqrt{2}c_s$. Abraham et al. (1994) have also observed propagation of transonic dislocations from a crack tip 9 in 6–12 Lenard–Jones solids.

Motivated by the experimental and numerical studies of transonic motion of cracks and dislocations, an attempt is made in this paper to provide a unified treatment of radiation-free transonic cracks and dislocations. We develop a mathematical framework in which the radiation-free cracks and dislocations can be studied simultaneously for comparison. We show that radiation-free transonic motion of a glide dislocation in an isotropic solid is indeed possible at $\sqrt{2}c_s$, rederiving the Eshelby theorem, and that such a critical speed does not apply to a climb dislocation, verifying the analysis of Weertman (1967). These results are then linked to radiation-free motion of Mode II shear cracks at $\sqrt{2}c_s$, which corresponds to a square-root singular crack tip stress field (Freund, 1979; Broberg, 1989). We use Stroh's method to generalize the Eshelby theorem to orthotropic and anisotropic elastic solids. In the case of orthotropic solids, the results agree with the analysis of asymptotic crack tip field of Huang et al. (1999) and agrees well with experimental observations of steady state transonic shear fracture in fiber-reinforced composites (Coker and Rosakis, 1998). We give a proof of existence of the radiation-free speed in the orthotropic case. In the case of general anisotropic solids, there are three wave speeds $c_3 < c_2 < c_1$ in any given crystal orientation at which a moving defect is considered. In the first transonic regime $c_3 < v < c_2$, we show that there always exists a radiation-free state for any given velocity of the defect. In the second transonic regime $c_2 < v < c_1$, the existence of radiation-free states appears to depend on both the symmetry properties of the material and the defect orientation. Examples of existence include a crack propagating in an isotropic solid and a crack propagating along a plane of symmetry in an orthotropic solid.

2. Transonic defects in isotropic solids

2.1. Supersonic radiation

It is known that supersonic motion of defects such as cracks and dislocations involves shock fronts, or traveling velocity discontinuity surfaces. These shock fronts transport energy away from the plane on which such defects move. The conventional wisdom is that steady-state motion in the supersonic regime cannot be maintained by externally applied loads. For dislocation motion, a possible way of achieving supersonic motion is by transforming the slip plane to a state of lower energy, as in the case of a partial dislocation removing a stacking fault (Hirth and Lothe, 1982).

A uniformly moving crack can be considered as consisting of a continuous array of dislocations moving at the same speed. The appearance of shock fronts at cracks and dislocations can be understood by examining the simplest case of a supersonic screw dislocation. In that case, the out-of-plane displacement satisfies the wave equation

$$c_s^2 \nabla^2 u_3 = \ddot{u}_3, \tag{1}$$

where ∇^2 is the Laplace operator, $\ddot{u}_3$ means derivative of u_3 with respect to time,

$$c_s = \sqrt{\frac{\mu}{\rho}} \tag{2}$$

is the shear wave speed, μ and ρ being the shear modulus and material density, respectively.

The solution to the above equation of motion for a screw dislocation moving at a supersonic speed $v > c_s$, is (Hirth and Lothe, 1982)

$$u_3 = \tfrac{1}{2} b H(c_s t - x_1 \cos\alpha - x_2 \sin\alpha), \quad x_2 > 0$$

$$u_3 = -\tfrac{1}{2} b H(c_s t - x_1 \cos\alpha + x_2 \sin\alpha), \quad x_2 < 0, \tag{3}$$

where $H(\ldots)$ is the Heaviside step function ($H(x) = 0$ when $x < 0$ and $H(x) = 1$ when $x > 0$) and the angle

$$\cos\alpha = c_s / v \tag{4}$$

determines the orientation at which the two diverging shock fronts move and carry energy away from the dislocation.

The above shock front solution characterizes the general feature of the deformation field of a dislocation or a crack (as an array of dislocations) in the supersonic regime. In the case of plane strain or plane stress deformation, the solution is usually more complicated due to the presence of an additional wave speed, the longitudinal wave speed

$$c_{\mathrm{d}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \tag{5}$$

where λ and μ are Lamé's constants. The longitudinal wave speed is roughly $\sqrt{3}$ times the shear wave speed c_{s} when the Poisson ratio is around 0.25. Three velocity regimes could be identified, with subsonic regime defined in the velocity range $0 < v < c_{\mathrm{s}}$, intersonic or transonic in the range $c_{\mathrm{s}} < v < c_{\mathrm{d}}$, and supersonic in the range $v > c_{\mathrm{d}}$. The transonic regime $c_{\mathrm{s}} < v < c_{\mathrm{d}}$ is of special interest because there is a supersonic component of the solution with respect to the shear wave and a subsonic component with respect to the longitudinal wave. A transonic defect moves with shock fronts characterized by the shear wave speed and is also subjected to a subsonic stress field characterized by the longitudinal wave speed.

2.2. The mathematical solution of a radiation-free transonic defect

In two-dimensional elastodynamics, the in-plane displacements are often represented in terms of two potential functions ϕ and ψ as

$$u_1 = \phi_{,1} + \psi_{,2}, \quad u_2 = \phi_{,2} - \psi_{,1}, \tag{6}$$

where comma indicates differentiation (e.g., $\phi_{,1} = \partial\phi/\partial x_1$). Combining these equations with equilibrium equations indicates

$$c_{\mathrm{d}}^2 \nabla^2 \phi = \ddot{\phi}, \quad c_{\mathrm{s}}^2 \nabla^2 \psi = \ddot{\psi}. \tag{7}$$

Therefore, the potential ϕ represents the part of the displacement field associated with the longitudinal wave and ψ represents that associated with the shear wave. In the transonic velocity regime, ϕ remains subsonic while ψ takes the form of shock front solutions similar to (3).

We are interested in the question whether there exists a radiation-free, purely subsonic solution for a crack or a dislocation moving in the transonic velocity regime. One approach to this question is to derive the full solution of a moving defect and examine the conditions under which the supersonic components vanish. An alternative approach is to probe the existence of a purely subsonic solution in the transonic regime. The equivalence of the two approaches are guaranteed by the uniqueness theorem of elasticity. The latter approach is more convenient for our present purpose and will be adopted here.

We seek purely subsonic solutions for a defect moving at a speed in the transonic regime $c_s < v < c_d$. This class of solutions exists only if the supersonic component ψ exactly vanish, in which case the displacement field becomes irrotational,

$$u_k = \phi_{,k}. \tag{8}$$

Each component of the displacement vector independently satisfies the wave equation

$$c_d^2 \nabla^2 u_k = \ddot{u}_k, \tag{9}$$

which under the steady-state condition can be rewritten as

$$\alpha_d^2 u_{k,11} + u_{k,22} = 0, \tag{10}$$

where

$$\alpha_d^2 = 1 - \frac{v^2}{c_d^2}. \tag{11}$$

The general form for a possible solution is

$$u_i = \text{Re}\,[A_i f(x_1 - vt + i\alpha_d x_2)], \tag{12}$$

where Re stands for the real part of a complex variable, and A_i are two arbitrary complex constants that are linked by the relation

$$A_2 = i\alpha_d A_1 \tag{13}$$

due to the irrotationality of displacements. Here we have taken $f(z) = \bar{f}(z)$, which is a valid assumption for crack and dislocation problems to be considered in this paper.

The stress field associated with the radiation-free transonic defect is

$$\sigma_{11} = \rho(c_d^2 \nabla^2 \phi - 2c_s^2 \phi_{,22}) = \rho(v^2 \phi_{,11} - 2c_s^2 \phi_{,22})$$

$$\sigma_{22} = \rho(c_d^2 \nabla^2 \phi - 2c_s^2 \phi_{,11}) = \rho(v^2 - 2c_s^2)\phi_{,11}$$

$$\sigma_{12} = 2\rho c_s^2 \phi_{,12}. \tag{14}$$

A quantity that will be convenient for our subsequent discussions on transonic dislocations is the resultant body force on the material within a closed contour surrounding the dislocation. Following Stroh (1962), let c be such a closed contour surrounding the dislocation and moving with it without change of shape. The stresses contribute a resultant force on the material inside c of

$$F'_i = \oint_c \sigma_{ij} n_j \, \mathrm{d}s, \tag{15}$$

where n_j is the unit normal to c. The rate at which momentum flows into c is

$$F''_i = \oint_c \rho \dot{u}_i v n_1 \, \mathrm{d}s. \tag{16}$$

In a steady state, the net change of linear and angular momentum inside c must be zero. Conservation of linear momentum indicates that the resultant body force F_i is

$$F_i = -F'_i - F''_i. \tag{17}$$

Substituting (14) and (8) into (15)–(17) yields the resultant forces

$$F_1 = 2\rho c_\mathrm{s}^2 \int_C \frac{\partial \phi_{,2}}{\partial s} \, \mathrm{d}s = 2\rho c_\mathrm{s}^2 [\![\phi_{,2}]\!],$$

$$F_2 = \rho(v^2 - 2c_\mathrm{s}^2) \int_C \frac{\partial \phi_{,1}}{\partial s} \, \mathrm{d}s = \rho(v^2 - 2c_\mathrm{s}^2) [\![\phi_{,1}]\!]. \tag{18}$$

The notation $[\![\dots]\!]$ is used to denote the jump in a multivalued function along the closed contour c.

2.3. The radiation-free transonic crack

Previous studies on transonic fracture (Freund, 1979; Broberg, 1989) in isotropic solids have emphasized on the asymptotic nature of the crack tip field. Here we will examine the transonic crack from a slightly different perspective by seeking solutions of a purely subsonic nature in the transonic regime. This will allow us to form a unified view on the radiation-free motion of both cracks and dislocations.

We seek purely subsonic solutions for a crack moving at a speed in the transonic regime $c_\mathrm{s} < v < c_\mathrm{d}$. Following the framework of Section 2.2, a possible solution is

$$u_i = \mathrm{Re}\left[A_i \sqrt{x_1 - vt + i\alpha_\mathrm{d} x_2} \right], \tag{19}$$

where A_i are complex constants linked by (13). The stresses can be derived from the displacements by (14) as

$$\sigma_{11} = \rho(v^2 u_{1,1} - 2c_\mathrm{s}^2 u_{2,2}),$$

$$\sigma_{22} = \rho(v^2 - 2c_\mathrm{s}^2) u_{1,1},$$

$$\sigma_{12} = 2\rho c_s^2 u_{2,1}. \tag{20}$$

Obviously, there exists no solution for a radiation-free Mode I crack in $c_s < v < c_d$, which can be seen from the fact that both A_1 and A_2 have to vanish in order to satisfy the symmetry condition $\sigma_{12} = 0$ on the crack plane.

In the case of a Mode II crack, the symmetry condition $\sigma_{22} = 0$ along the crack plane is satisfied at the critical speed $v_{rf} = \sqrt{2}c_s$. The traction-free condition along the crack face is satisfied if we choose the complex constants A_1 and A_2 as

$$A_2 = \frac{K}{\rho c_s^2 \sqrt{2\pi}}, \quad A_1 = \frac{K}{i\rho c_s^2 \alpha_d \sqrt{2\pi}}, \tag{21}$$

where K is the stress intensity factor defined such that the shear stress ahead of the crack tip is

$$\sigma_{12} = \frac{K}{\sqrt{2\pi(x_1 - vt)}}. \tag{22}$$

Correspondingly, the relative crack surface displacements are

$$\Delta u_1 = \frac{2K}{\rho c_s^2 \alpha_d \sqrt{2\pi}} \sqrt{vt - x_1}, \quad \Delta u_2 = 0. \tag{23}$$

The usual crack closure integral leads to the relationship between the energy release rate G and stress intensity factor K as

$$G = \frac{K^2}{4\rho c_s^2 \alpha_d} = \frac{\beta K^2}{E}, \tag{24}$$

where

$$\beta = \frac{1+v}{2}\sqrt{\frac{1-v}{v}}, \tag{25}$$

which is around 1.08 when $v = 0.25$.

The above results indicate that radiating shock fronts disappear when a crack under Mode II loading moves exactly at the speed $\sqrt{2}c_s$. The implication is that a Mode II crack can achieve a steady state motion at $\sqrt{2}c_s$. From the path-independent property of the J-integral, a radiation-free crack must have an inverse-square-root singularity, which confirms previous asymptotic analysis of transonic cracks (e.g., Freund, 1979; Broberg, 1989).

2.4. The radiation-free transonic dislocation

Rapid propagation of dislocations beyond subsonic regime has also been investigated in the past (Weertman, 1967, 1969; Hirth and Lothe, 1982). Eshelby (1949) first noted that the character of a transonic gliding edge dislocation becomes purely subsonic at a particular speed $\sqrt{2}c_s$. In other words, the radiating shock fronts disappear when a gliding edge dislocation moves exactly at the speed $\sqrt{2}c_s$. If a Mode II shear crack is considered as a continuous array of gliding edge dislocations, the Eshelby result implies that a Mode II shear crack propagating at $\sqrt{2}c_s$ is free of radiation and must have the inverse-square-root singularity. A unified view of transonic defects is that the critical speed $\sqrt{2}c_s$ represents an isolated radiation-free state within the transonic velocity regime.

The radiation-free dislocation problem could be analyzed in the same manner as the radiation-free crack. Let us seek purely subsonic solutions for an edge dislocation moving at a transonic speed $c_s < v < c_d$. In order to see the mathematical connection with the transonic crack analysis, we shall not impose the restriction that the dislocation be of glide type, as in the analysis of Eshelby (1949). Rather we consider an arbitrary Burgers vector even though it may be physically impossible for a climb type edge dislocation to propagate near elastic wave speeds. The general form of the dislocation solution we seek is

$$u_i = \mathrm{Re}[A_i \ln(x_1 - vt + i\alpha_d x_2)] \tag{26}$$

where the complex constants A_i are related to each other by (13).

For a uniformly moving dislocation, there exist no resultant forces on the material inside a closed contour c surrounding the dislocation. For the dislocation problem, we note that $[\![\phi_{,1}]\!]$ and $[\![\phi_{,2}]\!]$ define the components of Burgers vector,

$$[\![\phi_{,1}]\!] = b_1, \quad [\![\phi_{,2}]\!] = b_2. \tag{27}$$

From (18), for non-trivial solutions, the zero resultant force condition gives

$$[\![\phi_{,2}]\!] = b_2 = 0, \quad v = \sqrt{2}c_s. \tag{28}$$

Thus the only non-trivial solution for a radiation-free transonic dislocation is a glide edge dislocation with $b_1 \neq 0$ moving at $\sqrt{2}c_s$. A transonic Mode I crack can be represented as an array of climb-type edge dislocations, hence no radiation-free state can be found. On the other hand, a steady state transonic Mode II shear crack can be represented as an array of gliding edge dislocations moving at the same speed. This perspective provides a unified view of radiation-free transonic motion of cracks and dislocations.

The radiation-free solution of a transonic glide edge dislocation is simply given by (26) with

$$A_1 = \frac{b_1}{2\pi i}, \quad A_2 = \frac{\alpha_d b_1}{2\pi}. \tag{29}$$

3. Transonic defects in anisotropic solids

The discussions in Section 2 on radiation-free transonic cracks and dislocations in the isotropic elastic solid can be generalized to anisotropic elastic solids where there exist in general three different bulk wave speeds $c_3 \le c_2 \le c_1$ in any given crystal orientation at which a moving defect is considered (Stroh, 1962). When these wave speeds are distinct, there are four different velocity regimes: subsonic regime with the speed of the defect in the velocity range $0 < v < c_3$, first transonic regime with $c_3 < v < c_2$, second transonic regime with $c_2 < v < c_1$, and supersonic regime with $v > c_1$. While subsonic dislocations have been discussed by Stroh (1962), and the general features of transonic and supersonic dislocations are reasonably well understood, possible existence of radiation-free transonic dislocations has not been investigated previously.

3.1. Stroh's method

We will follow the method developed by Stroh (1962) for steady state dislocation motion in an anisotropic elastic solid. For steady state motion

$$u_i = u_i(x_1 - vt, x_2), \quad \sigma_{ij} = \sigma_{ij}(x_1 - vt, x_2), \tag{30}$$

the displacement solutions can be written in the form

$$u_i = \mathrm{Re}[A_i f(x_1 - vt + px_2)], \tag{31}$$

where A_i is the eigenvector, and the constant p is the Stroh eigenvalue which is determined as follows. Substituting (31) into the Navier equation of motion

$$C_{ijkl}\, \partial^2 u_k / \partial x_j\, \partial x_l = \rho\, \partial^2 u_i / \partial t^2, \tag{32}$$

where C_{ijkl} is the elastic modulus tensor, we arrive at the algebraic equation (Stroh, 1962; Eshelby et al., 1953)

$$(C_{i1k1} - \rho v^2 \delta_{ik} + pC_{i1k2} + pC_{i2k1} + p^2 C_{i2k2})A_k = 0. \tag{33}$$

For a non-trivial solution of A_k, the determinant of the coefficient matrix must vanish

$$\| C_{i1k1} - \rho v^2 \delta_{ik} + pC_{i1k2} + pC_{i2k1} + p^2 C_{i2k2} \| = 0. \tag{34}$$

This sextic equation produces six eigenvalues. In the subsonic regime $v < c_3$, all the eigenvalues are complex and form three complex conjugate pairs. In the first transonic regime $c_3 < v < c_2$, two of the eigenvalues become real and generate a pair of diverging shock fronts at a moving defect. In the second transonic regime $c_2 < v < c_1$, four of the eigenvalues become real and generate two pairs of diverging shock fronts at the defect. In the supersonic regime $v > c_1$, all eigenvalues are real and there is no longer a subsonic component to the solution. Following Stroh (1962), we order and label the eigenvalues $p_{\pm\alpha}$ ($\alpha = 1, 2, 3$) such that $p_{\pm\alpha}$ becomes real when $v \ge c_\alpha$. We further order the eigenvalues in such a way that, if complex valued, p_α has positive imaginary part while $p_{-\alpha}$ has negative imaginary part.

The stresses are represented in terms of three stress functions

$$\phi_i = \mathrm{Re}\big[L_i f(x_1 - vt + px_2)\big] \tag{35}$$

($i = 1, 2, 3$) as

$$\sigma_{i1} = -\phi_{i,2} - \rho \dot{u}_i v, \quad \sigma_{i2} = \phi_{i,1}. \tag{36}$$

After the displacement eigenvector A_k is found, Hooke's law relates the stress eigenvector L_i to A_k as

$$L_i = (C_{i2k1} + pC_{i2k2})A_k. \tag{37}$$

Similar to the calculations shown in Section 2, the resultant forces on the material inside a closed contour c can be calculated. The result is (Stroh, 1962)

$$F_i = [\![\phi_i]\!]. \tag{38}$$

3.2. Transonic cracks in orthotropic solids

Motivated by recent experiments (Coker and Rosakis, 1998; Liu et al., 1999) and theory (Huang et al., 1999) on dynamic crack propagation in fiber reinforced composites with orthotropic symmetry, we first study a transonic crack propagating along a symmetry plane (the x_1-direction) of an orthotropic solid. In this case, the in-plane and out-of-plane deformation decouple from each other. The Stroh eigenvalues governing the in-plane deformation are determined from

$$\begin{vmatrix} C_{11} - \rho v^2 + p^2 C_{66} & p(C_{12} + C_{66}) \\ p(C_{12} + C_{66}) & C_{66} - \rho v^2 + p^2 C_{22} \end{vmatrix} = 0, \tag{39}$$

where, following the convention, the fourth-order elastic moduli C_{ijkl} is condensed to a 6×6 matrix C_{pq} following the subscript conversion rule $(11) \to 1$, $(22) \to 2$, $(33) \to 3$, (23 or 32) $\to 4$, (13 or 31) $\to 5$, (12 or 21) $\to 6$ (e.g., $C_{1121} = C_{16}$). The two wave speeds in the direction of x_1 associated with in-plane deformation are

$$c_1 = \sqrt{\frac{C_{11}}{\rho}}, \quad c_2 = \sqrt{\frac{C_{66}}{\rho}}. \tag{40}$$

In the transonic regime, the solution to (39) is

$$p_{\pm 1} = \pm i\beta_1, \quad p_{\pm 2} = \pm \beta_2 \tag{41}$$

where β_1 and β_2 are real and positive constants given by (Huang et al., 1999)

$$\beta_1 = \sqrt{\frac{\sqrt{B^2+4AC}+B}{2A}}, \quad \beta_2 = \sqrt{\frac{\sqrt{B^2+4AC}-B}{2A}}$$

$$A = C_{22}C_{66}$$

$$B = C_{11}C_{22} - (C_{22}+C_{66})\rho v^2 - C_{12}^2 - 2C_{12}C_{66}$$

$$C = (C_{11} - \rho v^2)(\rho v^2 - C_{66}). \tag{42}$$

We seek radiation-free solutions of a purely subsonic nature. By the path-independence property of the J-integral, such solutions, if they exist, must have an inverse-square-root singularity near the crack tip. We thus write

$$u_i = \mathrm{Re}\left[A_i\sqrt{x_1 - vt + p_1x_2}\right]$$

$$\phi_i = \mathrm{Re}\left[L_i\sqrt{x_1 - vt + p_1x_2}\right] \tag{43}$$

where the eigenvector A_i associated with the eigenvalue p_1 is determined from

$$(C_{11} - \rho v^2 + p_1^2C_{66})A_1 + p_1(C_{12}+C_{66})A_2 = 0$$

$$p_1(C_{12}+C_{66})A_1 + (C_{66} - \rho v^2 + p_1^2C_{22})A_2 = 0 \tag{44}$$

to an arbitrary complex constant. Hooke's law, Eq. (37), becomes in the present case,

$$L_1 = p_1C_{66}A_1 + C_{66}A_2, \quad L_2 = C_{12}A_1 + p_1C_{22}A_2. \tag{45}$$

The two equations above can be combined to give a useful relation

$$L_1 + L_2p_1 = \rho v^2 A_2. \tag{46}$$

For a Mode I crack, symmetry requires that σ_{12} vanish along the entire plane of the crack. This leads to

$$L_1 = 0. \tag{47}$$

The traction-free condition on the crack surface requires $\sigma_{22}=0$ there, which gives

$$\mathrm{Re}[iL_2] = 0, \tag{48}$$

indicating that L_2 must be real. Combining (45) and (47), we obtain

$$A_2 = -p_1A_1 \tag{49}$$

for Mode I. The second equation of the eigenvalue problem (44) then gives

$$(C_{22}p_1^2 - C_{12} - \rho v^2)A_2 = 0. \tag{50}$$

The coefficient of A_2 in the above parentheses is always negative in the entire transonic regime since $p_1^2 = -\beta_1^2 < 0$. The only solution is $A_2 = 0$, as well as $A_1 = 0$ from (49). Therefore, there is no radiation-free state of a transonic crack under Mode I conditions.

For a Mode II crack, symmetry requires that σ_{22} vanishes along the entire plane of the crack. This leads to

$$L_2 = 0. \tag{51}$$

The traction-free condition on the crack surface requires $\sigma_{12} = 0$ there, corresponding to

$$\mathrm{Re}[iL_1] = 0, \tag{52}$$

i.e., L_1 must be real. Inserting (51) into (45) leads to

$$A_1 = -p_1 \frac{C_{22}}{C_{12}} A_2 \tag{53}$$

for Mode II. The second equation of the eigenvalue problem (44) then gives

$$f_{\mathrm{II}}(v)A_2 = 0, \tag{54}$$

where

$$f_{\mathrm{II}}(v) = p_1^2 C_{22}C_{66} - C_{12}(C_{66} - \rho v^2). \tag{55}$$

To have a non-trivial solution for A_2, as well as for A_1 in (53), the coefficient f_{II} must vanish. Therefore, the critical condition for a radiation-free Mode II crack is thus

$$f_{\mathrm{II}}(v) = -\beta_1^2 C_{22}C_{66} - C_{12}(C_{66} - \rho v^2) = 0. \tag{56}$$

In contrast to the Mode I case, there always exists a solution for v within the transonic regime $c_2 < v < c_1$ for a Mode II crack because

$$f_{\mathrm{II}}(c_1) = C_{12}\rho(c_1^2 - c_2^2) > 0, \quad f_{\mathrm{II}}(c_2) = -\beta_1^2 C_{22}C_{66} < 0. \tag{57}$$

Solving Eq. (56) with (42) gives the radiation-free speed

$$v_{\mathrm{rf}} = \sqrt{\frac{C_{11}C_{22} - C_{12}^2}{\rho(C_{12} + C_{22})}}. \tag{58}$$

This speed was first obtained by Huang et al. (1999) who studied the critical speed at which the singularity near the tip of a transonic crack becomes $-1/2$. For the Graphite/Epoxy unidirection fiber-reinforced composite, the calculated radiation-free speed is about $v_{\mathrm{rf}} = 4.532c_2 = 8700$ m/s, which is compared to the experimentally observed stable crack tip velocity of approximately 8000 m/s. The difference can be attributed to the uncertainties in elastic constants of these composites and possibly also to non-linear interaction between the shock generation and the fracture process near the crack tip.

The above analysis indicates that a radiation-free transonic crack in orthotropic solids must be Mode II, similar to the isotropic case. Defining the stress intensity factor K such that the shear stress ahead of the crack tip is

$$\sigma_{12} = \frac{K}{\sqrt{2\pi(x_1 - vt)}}. \tag{59}$$

It follows that for the present case,

$$L_1 = \frac{2K}{\sqrt{2\pi}}, \quad A_1 = -\frac{2p_1 C_{22} K}{\rho v^2 C_{12}\sqrt{2\pi}}, \quad A_2 = \frac{2K}{\rho v^2 \sqrt{2\pi}}. \tag{60}$$

The relative crack surface displacements are

$$\Delta u_1 = \frac{4\beta_1 C_{22} K}{\rho v_{\rm rf}^2 C_{12}\sqrt{2\pi}}\sqrt{vt - x_1}, \quad \Delta u_2 = 0. \tag{61}$$

The usual crack closure integral leads to the energy release rate

$$G = \frac{\beta_1 C_{22} K^2}{2\rho v_{\rm rf}^2 C_{12}}. \tag{62}$$

3.3. Transonic dislocations in orthotropic solids

We now show that the radiation-free speed of a transonic dislocation propagating along a symmetry plane of an orthotropic solid is identical to that of a crack.

We seek possible radiation-free solutions of a transonic dislocation of Burgers vector b_i ($i=1, 2$) and write

$$u_i = \mathrm{Re}[A_i \ln(x_1 - vt + p_1 x_2)],$$

$$\phi_i = \mathrm{Re}[L_i \ln(x_1 - vt + p_1 x_2)], \tag{63}$$

where A_i is related to the Burgers vector b_i by

$$b_i = \mathrm{Re}[2\pi i A_i]. \tag{64}$$

There are no resultant forces on the material inside a closed contour around the dislocation. Inserting (63) into (38) leads to

$$\mathrm{Re}[iL_k] = 0. \tag{65}$$

For a climb edge dislocation (i.e., $b_1 = 0$), the shear stress σ_{12} vanishes along the plane of motion due to symmetry, which when combined with (65) requires

$$L_1 = 0, \quad \mathrm{Re}[iL_2] = 0. \tag{66}$$

These are identical to the conditions given in (47) and (48) on the eigenvector L_i for the radiation-free Mode I crack. Following the same analysis given in (49) and (50) leads to the conclusion that there is no radiation-free state of a transonic climb edge dislocation.

For a glide edge dislocation (i.e., $b_2 = 0$), the normal stress σ_{22} vanishes along the plane of motion due to symmetry, which when combined with (65) requires

$$L_2 = 0, \quad \mathrm{Re}[iL_1] = 0. \tag{67}$$

These are identical to the conditions given in (51) and (52) on the eigenvector L_i for the radiation-free Mode II crack. Therefore, following the same steps from (53)–(58) yields a radiation-free speed of a transonic dislocation identical to that of a transonic crack given in (58).

As has been discussed in the isotropic case, the connection between a climb edge dislocation and a Mode I crack, and that between a glide edge dislocation and a Mode II shear crack are not surprising because the crack can be modeled as a continuous array of dislocations moving at the same speed.

3.4. Radiation-free defects in general anisotropic solids

We now attempt to generalize the analysis of radiation-free transonic defects to the case of general anisotropic solids where there exist in general three different bulk wave speeds $c_3 < c_2 < c_1$ in any given crystal orientation at which a moving defect is considered (Stroh, 1962). We consider separately the case of first transonic regime $c_3 < v < c_2$ and second transonic regime $c_2 < v < c_1$. We will discuss a radiation-free transonic dislocation and a radiation-free transonic crack in the same setting.

In the first transonic regime, the eigenvalues $p_{\pm 3}$ are real while $p_{\pm 1}$ and $p_{\pm 2}$ are complex valued. We seek purely subsonic solutions where the supersonic component relating to $p_{\pm 3}$ vanish. In that case, the displacement field and the stress functions for a radiation-free transonic dislocation are written in the form,

$$u_k = \mathrm{Re}[A_{k1} \ln(z_1) + A_{k2} \ln(z_2)], \quad \phi_k = \mathrm{Re}[L_{k1} \ln(z_1) + L_{k2} \ln(z_2)], \tag{68}$$

where $z_\alpha = x_1 - vt + p_\alpha x_2$. Similarly, the corresponding fields for a radiation-free transonic crack are

$$u_k = \mathrm{Re}\left[A_{k1}\sqrt{z_1} + A_{k2}\sqrt{z_2}\right], \quad \phi_k = \mathrm{Re}\left[L_{k1}\sqrt{z_1} + L_{k2}\sqrt{z_2}\right]. \tag{69}$$

We do not normalize the eigenvectors so that arbitrary constant coefficients are not necessary in the above superposition.

Requiring that the tractions on the upper and lower surfaces of a crack vanish or that the resultant forces on the material inside a closed contour around a dislocation vanish, we obtain

$$\mathrm{Re}[i(L_{k1} + L_{k2})] = 0. \tag{70}$$

These are three conditions for $k = 1, 2, 3$.

According to (33), the two displacement eigenvectors $A_{k\alpha}$ ($\alpha = 1, 2$) are determined from the algebraic equation

$$(C_{i1k1} - \rho v^2 \delta_{ik} + p_\alpha C_{i1k2} + p_\alpha C_{i2k1} + p_\alpha^2 C_{i2k2}) A_{k\alpha} = 0 \tag{71}$$

to two arbitrary complex constants, one for A_{k1} and one for A_{k2}. According to (37), the two stress eigenvectors $L_{k\alpha}$ are related to $A_{k\alpha}$ by

$$L_{i\alpha} = (C_{i2k1} + p_\alpha C_{i2k2}) A_{k\alpha}. \tag{72}$$

Therefore, the mathematical problem of the existence of radiation-free states in the first transonic regime is reduced to having four unknown real constants (two complex constants in the two displacement eigenvectors) satisfy three real equations given in (70). Since this could be posed as a 4×4 homogeneous algebraic equation problem with one null equation, the determinant of the coefficient matrix is always zero and a non-trivial solution can always be found for any given velocity of a moving defect. Therefore, we have arrived at the following theorem for radiation-free transonic defects:

Theorem. *At any given velocity in the first transonic regime, there always exists a radiation-free transonic state of a moving defect in an anisotropic solid.*

In the relatively familiar subsonic regime, a crack moves without radiation with arbitrary mixity of fracture Modes I, II and III; a dislocation moves without radiation with arbitrary Burgers vector. In the first transonic regime, although it is always possible to find a radiation-free state, the mode mixity of a crack or the Burgers vector of a dislocation is constrained. This can be seen from the fact that there are three constraint equations in (70) on four degrees of freedom from the displacement eigenvectors. Consider a trivial example of an orthotropic solid where there are two wave speeds associated with in-plane deformation and one wave speed associated with out-of-plane deformation. Without loss of generality, we label the in-plane wave speeds as c_2 and c_1, with $c_2 < c_1$, and the out-of-plane wave speed c_3. If $c_3 < c_2$, the first transonic regime $c_3 < v < c_2$ corresponds to the subsonic motion for the in-plane deformation so that a mixed Mode I/II crack or a mixed glide/climb edge dislocation moves without radiation. If $c_2 < c_3$, the first transonic regime $c_2 < v < c_3$ changes to the subsonic motion for the out-of-plane deformation so that a Mode III crack or a screw dislocation moves without radiation. From this example, we see that the fracture mode or the Burgers vector cannot be arbitrarily chosen for the radiation-free state in the first transonic regime.

For the second transonic regime, two pairs of eigenvalues $p_{\pm 2}$ and $p_{\pm 3}$ are real while $p_{\pm 1}$ are complex valued. We seek subsonic solutions by writing the displacement field and the stress functions for a radiation-free transonic dislocation in the form,

$$u_k = \mathrm{Re}[A_k \ln(z_1)], \quad \phi_k = \mathrm{Re}[L_k \ln(z_1)]. \tag{73}$$

Similarly, the corresponding fields for a radiation-free transonic crack are

$$u_k = \mathrm{Re}\big[A_k\sqrt{z_1}\big], \quad \phi_k = \mathrm{Re}\big[[L_k\sqrt{z_1}\big]. \tag{74}$$

The condition of no crack face traction or no spurious forces on a dislocation requires

$$\mathrm{Re}[iL_k] = 0, \tag{75}$$

which gives three real equations for $k=1$, 2, 3. According to (33), the displacement eigenvector A_k is determined from the algebraic equation

$$(C_{i1k1} - \rho v^2\delta_{ik} + p_1 C_{i1k2} + p_1 C_{i2k1} + p_1^2 C_{i2k2})A_k = 0 \tag{76}$$

to an arbitrary complex constant. According to (37), the stress eigenvector L_k is related to A_k by

$$L_i = (C_{i2k1} + p_1 C_{i2k2})A_k. \tag{77}$$

Therefore, the mathematical problem of the existence of radiation-free states in the second transonic regime is reduced to having two unknown real constants (one complex constant in the displacement eigenvector) satisfy three real equations given in (75). In this case, a non-trivial solution cannot be guaranteed unless significant symmetry conditions exist. For example, as soon as the in-plane deformation is decoupled from the out-of-plane deformation, the existence of radiation-free states in the second transonic regime could be posed as a homogeneous algebraic problem of finding two unknown real constants in the displacement eigenvector which satisfy two real equations given in (75) for $k=1$, 2. A radiation-free speed can then be determined by requiring that the determinant of the coefficient matrix vanish. The radiation-free speed $\sqrt{2}c_s$ for an isotropic solid and that of (58) for an orthotropic solid correspond to the isolated radiation-free state in the second transonic regime for anisotropic solids of special symmetry. In general, the existence of such states depends on both the crystal symmetry of the solid and the defect orientation. Further case studies of various classes of anisotropic solids in the second transonic regime are deferred to future work on this subject.

4. Summary

In this paper, we have achieved the following:

1. A mathematical framework is developed in which the radiation-free transonic cracks and dislocations can be studied simultaneously for comparison. Instead of deriving the full transonic solutions and investigating the conditions under which the supersonic components of the solutions vanish, we probe the existence of radiation-free solutions in the transonic regime. The equivalence of these two approaches is guaranteed by the uniqueness theorem of elasticity.
2. We have attempted to provide a unified treatment of transonic motion of dislocations and cracks. We showed that radiation-free transonic motion of a glide dislocation in an isotropic solid is indeed possible at $\sqrt{2}c_s$, rederiving the Eshelby theorem, and that such a critical speed does not apply to a climb dislocation, verifying the analysis of Weertman (1967). These results are then linked to radiation-free motion of Mode II shear cracks at $\sqrt{2}c_s$, which corresponds to a square-root singular crack tip stress field (Freund, 1979; Broberg, 1989).
3. Stroh's method of analyzing steady-state motion of defects has been used to generalize the Eshelby theorem of radiation-free transonic dislocation to orthotropic and anisotropic elastic solids. In the case of orthotropic solids, our results agree with the analysis of asymptotic crack tip field of Huang et al. (1999) and agrees well with experimental observations of steady state transonic shear fracture in fiber-reinforced composites (Coker and Rosakis, 1998). We have given a proof of existence of the radiation-free speed in the orthotropic case.
4. For general anisotropic elastic solids, there are in general three wave speeds $c_3 < c_2 < c_1$ in any given crystal orientation at which a moving defect is considered. In the first transonic regime $c_3 < v < c_2$, we have shown that there always exists a radiation-free state for any given velocity of a moving defect. In contrast to the subsonic regime where a crack of arbitrary mode mixity or a dislocation of arbitrary Burgers vector moves without radiation, cracks or dislocations in the first transonic regime must have a specific mode mixity or Burgers vector in order to move without radiation. In the second transonic regime $c_2 < v < c_1$, the radiation-free states usually do not exist unless significant symmetry conditions exist. Examples of existence include isotropic and orthotropic solids. Such states appear to depend on both the symmetry properties of the material and the defect orientation.

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References

Abraham, F.F., Brodbeck, D., Rafey, R.A., Rudge, W.E., 1994. Instability dynamics of fracture: a computer simulation investigation. Phys. Rev. Lett. 73, 272–275.

Archuleta, R.J., 1982. Analysis of near source static and dynamic measurements from the 1979 Imperial Valley earthquake. Bulletin of the Seismological Soc. of America 72, 1927–1956.

Broberg, K.B., 1989. The near-tip field at high crack velocities. Int. J. Fracture 39, 1–13.

Burridge, R., 1973. Admissible speeds for plane-strain self-similar shear cracks with friction but lacking cohesion. Geophysical J. of the Royal Astronomical Soc. 35, 439–455.

Burridge, R., Conn, G., Freund, L.B., 1979. The stability of a plane strain shear crack with finite cohesive force running at intersonic speeds. J. of Geophysics Res. 84, 2210–2222.

Coker, D., Rosakis, A.J., 1998. Experimental observations of intersonic crack growth in asymmetrically loaded unidirectional composite plates. Caltech SM Report No. 98-16, California Institute of Technology, Pasadena, California, June, 1998.

Eshelby, J.D., 1949. Uniformly moving dislocations. Proc. Phys. Soc. 62A, 307–314.

Eshelby, J.D., Read, W.T., Shockley, W., 1953. Anisotropic elasticity with applications to dislocation theory. Acta Metall. 1, 251–259.

Freund, L.B., 1979. The mechanics of dynamic shear crack propagation. J. of Geophysics Res. 84, 2199–2209.

Freund, L.B., 1990. Dynamic fracture mechanics. Cambridge University Press, Cambridge, UK.

Gao, H., 1993. Surface roughening and branching instabilities in dynamic fracture. J. Mech. Phys. Solids 41, 457–486.

Georgiadis, H.G., 1986. On the stress singularity in steady-state transonic shear crack propagation. Int. J. Frac. 30, 175–180.

Gumbsch, P., Gao, H., 1999 Dislocations faster than the speed of sound, Science 283, 965–968.

Hirth, J.P., Lothe, J., 1982. Theory of dislocations. John Wiley and Sons, New York Chapter 7.

Hoover, W.G., Hoover, N.E., Moss, W.C., 1977. Steady state dislocation motion via molecular dynamics. Phys. Lett. A 63A, 324–326.

Huang, Y., Wei, W., Liu, C., Rosakis, A.J., 1999. Analysis of intersonic crack growth in unidirectional fiber-reinforced composites. J. Mech. Phys. Solids (in press).

Liu, C., Rosakis, A.J., Stout, M., Lovato, M., Ellis, R. 1999. On the application of CGS interferometry to the study of mode-I dynamic crack growth in unidirectional composites (in preparation).

Payton, R.C., 1989. Transient stresses in a transversely isotropic solid caused by a dislocation moving at tran- or super-sonic speeds. Z. Angew. Math. Phys. 40, 133–138.

Payton, R.C., 1995. Steady state stresses induced in a transversely isotropic elastic solid by a moving dislocation. Z. Angew. Math. Phys. 46, 282–288.

Rosakis, A.J., Coker, D., Samundrala, O., 1998. Shear dominated intersonic crack growth in weakly bonded Homalite-100 plates. Caltech SM Report No. 98-17, California Institute of Technology, Pasadena, California, June, 1998.

Simonov, I.V., 1983. Behavior of solutions of dynamic problems in the neighborhood of the edge of a cut moving at transonic speed in an elastic medium. Mechanics of Solids (Mechanika Tverdogo Tela) 18, 100–106.

Slepyan, L.I., 1981. The problem of the propagation of a cut at transonic velocity. Soviet Phys. 26, 1192–1193.

Stroh, A.N., 1962. Steady state problems in anisotropic elasticity. J. Math. Phys. 41, 77–103.

Weertman, J., 1967. Uniformly moving transonic and supersonic dislocations. J. Appl. Phys. 38, 5293–5301.

Weertman, J., 1969. Dislocations in uniform motion on slip or climb planes having periodic force laws. In: Mura, T. (Ed.), Mathematical theory of dislocations. ASME, New York, pp. 178–202.

Zhang, Y.W., Wang, T.C., Tang, Q.H., 1995. Simulation of nucleation and emission of dislocations by molecular dynamic method. J. Appl. Phys. 77, 2393–2399.