PERGAMON

Journal of the Mechanics and Physics of Solids
51 (2003) 311 – 331

JOURNAL OF THE
MECHANICS AND
PHYSICS OF SOLIDS

www.elsevier.com/locate/jmps

Sudden deceleration or acceleration of an intersonic shear crack

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Received 31 January 2002; accepted 23 July 2002

Abstract

Sudden jumps in the crack tip velocity were revealed by numerical simulation (in both continuum/cohesive element and molecular dynamics approaches) and experiments for rapid shear cracking. The cracking velocity may accelerate from a sub-Rayleigh speed to the intersonic range, or from an intersonic speed to a higher one, when the reflected impact wave reloads the crack tip. On the other hand, the cracking velocity may decelerate from an intersonic speed to a lower one or recede to the sub-Rayleigh range when the fracture driving force declines. The velocity change encountered during intersonic cracking plays a different role from that in the acceleration or deceleration of a subsonic crack. A crack propagating at an intersonic speed would leave a shear wave trailing behind. When the crack decelerates or accelerates, the effect of the trailing wave will lead to a transition period from one steady-state solution of crack tip singularity to another. This investigation aims at quantifying these processes. The full field solution of an intersonic mode II crack whose speed changed suddenly from one velocity (intersonic or subsonic) to another (intersonic or subsonic) is given in closed form. The solution is facilitated via superposing a series of propagating crack problems that are loaded by dislocations to seal the unwanted crack-face sliding or by concentrated forces moving at various speeds to negate the crack-face traction. In contrast to the subsonic solution, the results in the intersonic case indicate that the elastic fields around the crack tip depend on the deceleration or acceleration history that is traced back over a long time. Singularity matching dictates the jump that may actually take place.

Keywords: A. Dynamic fracture; A. Stress intensity factor; B. Deceleration and acceleration; B. Elastic material; C. Integral transform

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0022-5096/03/$ - see front matter
PII: S0022-5096(02)00078-9

1. Introduction

The possibility of intersonic crack growth under shear-dominated loading was first examined by Burridge (1973) under the condition of negligible cohesion, with the crack tip velocity v between shear wave speed c_s and longitudinal wave speed c_l. Using a cohesive strip model, Andrews (1976) predicted that an intersonic crack eventually stabilizes at the crack tip velocity of $\sqrt{2}c_s$. The pioneering work on a steady-state intersonic crack ($c_s < v < c_l$) advancing in an elastic medium (Freund, 1979; Burridge et al., 1979) indicated that the stress singularity of intersonic cracks is less than $\frac{1}{2}$ except at a special (radiation free) crack tip velocity of $\sqrt{2}c_s$, at which the energy release rate is finite and non-vanishing. There were also early analytical studies of shear-dominated intersonic crack propagation at a constant crack tip velocity subjected to uniform shear on crack faces (Brock, 1977), stress singularity (Simonov, 1983; Broberg, 1989) and self-similar crack propagation (Broberg, 1994).

For the first time in laboratory experiments, Rosakis et al. (1999, 2000) observed shear-dominated intersonic crack propagation along a weak plane in an otherwise homogeneous brittle polyester resin. The crack tip velocity not only exceeded the widely believed limit for crack propagation, Rayleigh wave speed c_R, but also exceeded the shear wave speed c_s and approached the longitudinal wave speed c_l. The weak plane in polyester resin served as the preferred path for crack propagation, and prevented crack kinking and branching. Intersonic crack propagation was also observed in unidirectional fiber-reinforced composite materials (Coker and Rosakis, 2001), in which weak interfaces between the fibers and the matrix material became the preferred paths for crack propagation. Indirect evidence of shear crack propagation in excess of shear wave speed has also been provided from observations of shallow crustal earthquakes (Archuleta, 1982; Beroza and Spudich, 1988; Wald and Heaton, 1994; Ellsworth and Celebi, 1999).

Experiments by Rosakis et al. (1999, 2000) and Coker and Rosakis (2001) have motivated recent analytical and numerical studies on shear-dominated intersonic crack propagation. Broberg (1999a) and Huang et al. (1999) obtained the asymptotic crack tip field for an intersonic crack in an orthotropic solid, while Gao et al. (1999) investigated the radiation-free intersonic crack tip velocity in general anisotropic solids. Abraham and Gao (2000) used molecular dynamics to study transition of a subsonic shear crack to an intersonic crack propagating along the interface between two weakly bonded, identical harmonic crystals. Gao et al. (2001) studied the same problem with linear elasticity theory and established that, without any parameter fitting, continuum analysis agreed well with Abraham and Gao's (2000) atomistic simulations. Needleman (1999), Needleman and Rosakis (1999), Yu and Suo (2000), and Geubelle and Kubair (2001) investigated shear-dominated intersonic crack propagation with cohesive zone models in order to remedy the issue of vanishing crack tip energy release rate. Huang and Gao (2001) obtained the fundamental solution for an initially stationary crack starting to propagate at an intersonic speed once a pair of concentrated shear forces are applied on the crack faces. This fundamental solution can be used to construct the general solution for uniform intersonic crack propagation subjected to an arbitrary initial equilibrium field. Finally, Antipov and Willis (2002) obtained the fundamental solution for intersonic crack propagation in linear viscous solids.

For a crack propagating within the sub-Rayleigh regime ($v < c_R$), the solution for arbitrary, non-uniform crack tip motion has been obtained (e.g., Freund, 1972, 1990; Kostrov, 1975; Brock, 1977; Willis, 1989). Freund (1972) observed that, when a sub-Rayleigh crack is suddenly stopped, the equilibrium solution immediately radiates out from the crack tip at shear wave speed. This critical observation enabled him to obtain a solution for a crack that suddenly stops and resumes propagating at a different sub-Rayleigh velocity, and to construct a general solution for non-uniform (sub-Rayleigh) crack propagation. This eventually led to a universal relation between the dynamic stress intensity factor and its equilibrium counterpart for the same crack geometry. For intersonic crack propagation ($c_s < v < c_l$), however, recent work of Huang and Gao (2002) showed that the stress intensity factor does not instantaneously reach its equilibrium value when an intersonically propagating crack tip is suddenly arrested. This is because the Rayleigh and shear waves are still trailing behind the intersonic crack tip at the instant of crack arrest. In fact, Huang and Gao (2002) showed that the equilibrium stress intensity factor is reached after a finite delay, i.e. after all the waves trailing behind have passed the arrested crack tip. Accordingly, the universal relation between dynamic and static stress intensity factors does not hold for intersonic crack propagation.

Numerical simulation via continuum/cohesive elements (Needleman, 1999; Needleman and Rosakis, 1999) revealed sudden jumps in the crack tip velocity. When the impact wave arrives, the crack tip may accelerate from a sub-Rayleigh speed to an intersonic speed (typically close to the longitudinal wave speed) crossing the forbidden velocity zone, or from an intersonic speed to a higher one. The same phenomenon of instant acceleration was observed in molecular dynamics simulations (Abraham and Gao, 2000; Gao et al., 2001). The crack tip may also decelerate from an intersonic speed to a lower intersonic speed or to a sub-Rayleigh speed when the fracture driving force recedes. Experiments (Rosakis et al., 1999, 2000; Coker and Rosakis, 2001) have also suggested these sudden rises and falls of the shear cracking velocity. The velocity jump encountered during intersonic crack propagation plays a different role from that in the acceleration or deceleration of a subsonic crack. It may also serve as a crude estimate for the general non-uniform accelerating or decelerating motion of a shear crack within subsonic and intersonic ranges.

The above-mentioned points give us a strong impetus to investigate suddenly decelerating or accelerating intersonic cracks. Such a study would provide further insights into the fundamental difference between sub-Rayleigh and intersonic crack propagation. Unlike sub-Rayleigh crack propagation, the stress intensity factor for a crack propagating with a varying velocity in the intersonic range $[c_s, c_l]$ cannot be related to its equilibrium counterpart for the same crack geometry. This is because the Rayleigh and shear waves trailing behind will never catch up to pass the intersonic crack tip. Once the crack tip velocity drops below the Rayleigh wave speed, the trailing Rayleigh and shear waves will catch up with the sub-Rayleigh crack tip, and the universal relation (Freund, 1972) between static and sub-Rayleigh dynamic stress intensity factors for the same crack geometry will hold again.

We consider an infinite, linear elastic isotropic solid containing a semi-infinite crack under plane-strain deformation. A pair of concentrated shear forces are suddenly applied on crack faces, and at the same instant the crack tip starts to propagate at a constant velocity v_1 that can be sub-Rayleigh ($0 < v_1 < c_R$) or intersonic ($c_s < v_1 < c_l$). After propagating for a finite time t^*, the crack tip velocity suddenly changes to v_2 that can also be sub-Rayleigh or intersonic. A closed-form solution for a suddenly decelerating crack ($v_2 < v_1$) is obtained in Section 2 by superimposing a series of moving dislocation solutions to seal the sliding displacement between the decelerated crack tip and the crack tip that would continue to propagate at the initial velocity v_1. The case of a suddenly accelerating intersonic crack, which bears important differences from a decelerating crack, is solved in Section 3 by superimposing a series of moving point force solutions to negate the shear traction between the accelerated crack tip and the crack tip that would continue to propagate at the initial velocity v_1. Both analyses confirm Huang and Gao's (2002) observation that, unlike sub-Rayleigh crack propagation, stress intensity factor around an intersonically propagating crack tip depends on the crack velocity history.

2. Decelerating crack

Consider a crack occupying the negative half x-axis with crack tip located at the origin. At time $t = 0$, a pair of concentrated shear forces of magnitude τ^* is imposed at the origin, which drives the crack to extend along the positive x-axis at a velocity v_1. At a later time $t = t^*$, the crack suddenly decelerates to a lower velocity v_2 ($< v_1$) after an extension of $l = v_1 t^*$. This problem can be viewed as the superposition of following two sub-problems.

a. Continuous crack propagation at initial crack tip velocity v_1. This is the fundamental solution obtained by Huang and Gao (2001). One important characteristic of this solution is that the sliding displacement on crack faces ($x < 0, y = 0$) has the similarity form $u_x^F(x, y = 0, t) = u_x^F(x/t) = u_x^F(w)$, namely it depends on time t and coordinate x only through their ratio $w = x/t$.
b. Negation of crack-face sliding displacement between the actual crack tip, $x = v_1 t^* + v_2(t - t^*)$, and the crack tip that would continue to propagate at the initial velocity v_1, $x = v_1 t$, as stated in the first sub-problem above. This is achieved by emitting dislocations from the propagating crack tip in just the appropriate sequence. The moving dislocation solution has also been used in the study of a suddenly stopping intersonic crack (Huang and Gao, 2002).

2.1. A moving dislocation ahead of a propagating crack tip

The solution of a moving dislocation emitted from a stationary crack tip (Freund, 1990; Huang and Gao, 2002) is generalized for a propagating crack tip in this section for both sub-Rayleigh and intersonic crack propagation. An infinite solid containing a semi-infinite crack on the negative x-axis is stress free and at rest everywhere for time

$t < 0$. The crack tip begins to propagate from the origin (0, 0) along the positive x-axis at time $t = 0$, and the crack tip velocity is denoted by v_2. At the same instant $t = 0$, an edge dislocation is emitted from the crack tip and propagates along the positive x-axis at a higher velocity w ($> v_2$).

The in-plane displacements are expressed in terms of displacement potentials ϕ and ψ by

$$u_x = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad u_y = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}. \tag{2.1}$$

The in-plane stress components can be written in terms of ϕ and ψ as

$$\sigma_{xx} = \mu \left(\frac{c_1^2}{c_s^2} \nabla^2 \phi - 2 \frac{\partial^2 \phi}{\partial y^2} + 2 \frac{\partial^2 \psi}{\partial x \partial y} \right),$$

$$\sigma_{yy} = \mu \left(\frac{c_1^2}{c_s^2} \nabla^2 \phi - 2 \frac{\partial^2 \phi}{\partial x^2} - 2 \frac{\partial^2 \psi}{\partial x \partial y} \right),$$

$$\sigma_{xy} = \mu \left(2 \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right), \tag{2.2}$$

where ∇^2 denotes the Laplace operator, and μ the shear modulus.

The equations of motion are given by

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - \frac{1}{c_1^2} \frac{\partial^2 \phi}{\partial t^2} = 0, \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2} = 0. \tag{2.3}$$

The skew symmetry of the problem dictates that only the upper half-plane ($y \geqslant 0$) needs to be analyzed. The boundary conditions are

$$\sigma_{yy}(x, y = 0, t) = 0, \quad \sigma_{xy}(x < v_2 t, y = 0, t) = 0,$$

$$u_x(x > v_2 t, y = 0, t) = bH(wt - x), \tag{2.4}$$

where b is the (half) Burgers vector of the edge dislocation, and H is the unit step function. Since the analysis is rather similar to that of sub-Rayleigh (e.g., Freund, 1990) and intersonic (Huang and Gao, 2001) crack propagation subjected to a pair of concentrated forces, only the solution method and results pertinent to the decelerating crack problem are presented in the following.

A coordinate system $(\xi, y) = (x - v_2 t, y)$ moving with the crack tip is introduced. A double Laplace transform with respect to time t and moving coordinate ξ is applied to equations of motion (2.3) and boundary conditions (2.4). The Wiener–Hopf method of analytic continuation gives the solution in transformed space, and the Cagnaird–de Hoop method is then used to invert the double Laplace transform (e.g., Freund, 1990; Huang and Gao, 2001). Only the stress intensity factor k_{II} is given in the following for sub-Rayleigh and intersonic shear crack propagation.

2.1.1. Sub-Rayleigh shear crack propagation with a moving dislocation

The stress field has a square-root singularity around a crack tip propagating at a sub-Rayleigh velocity ($v_2 < c_R$). The stress intensity factor, defined by $k_{II} = \lim_{\xi \to 0^+} \sqrt{2\pi\xi}\sigma_{xy}(\xi, y = 0, t)$ for sub-Rayleigh crack propagation with a moving dislocation at velocity $w(v_2 < w < c_l)$, is given by

$$k_{II} = bk_0^{sub}(t, w, v_2), \tag{2.5}$$

where

$$k_0^{sub}(t, w, v_2) = -\mu \frac{\kappa_2 c_s^2}{\alpha_{s2} v_2^2} \frac{c_R + w}{c_R + v_2} \sqrt{\frac{c_s + v_2}{c_s + w}}\, s_-^{sub}\left(\frac{-1}{w - v_2}\right) \sqrt{\frac{2}{\pi(w - v_2)t}},$$

$$\kappa_2 = 4\alpha_{l2}\alpha_{s2} - (1 + \alpha_{s2}^2)^2, \quad \alpha_{l2} = \left(1 - \frac{v_2^2}{c_l^2}\right)^{1/2}, \quad \alpha_{s2} = \left(1 - \frac{v_2^2}{c_s^2}\right)^{1/2} \tag{2.6}$$

and

$$s_\pm^{sub}(\zeta) = \exp\left\{-\frac{1}{\pi}\int_{1/(c_l \mp v_2)}^{1/(c_s \mp v_2)} \tan^{-1}\left[\frac{4c_s^3 r^2 \sqrt{(1 - v_2 r)^2 - c_s^2 r^2}\sqrt{c_l^2 r^2 - (1 - v_2 r)^2}}{c_l[2c_s^2 r^2 - (1 \pm v_2 r)^2]^2}\right]\frac{dr}{r \pm \zeta}\right\}. \tag{2.7}$$

It can be verified that, as the crack tip velocity v_2 approaches zero, the above results (2.5)–(2.7) degenerate to Huang and Gao (2002) for a stationary crack tip with a dislocation moving at velocity w.

2.1.2. Intersonic shear crack propagation with a moving dislocation

The stress field around an intersonic shear crack tip ($c_s < v_2 < c_l$) has a singularity $\sigma \sim r^{-q_2}$, where r is the distance to the crack tip:

$$q_2 = \frac{1}{\pi}\tan^{-1}\frac{4\alpha_{l2}\hat{\alpha}_{s2}}{(2 - v_2^2/c_s^2)^2}, \tag{2.8}$$

$$\alpha_{l2} = \left(1 - \frac{v_2^2}{c_l^2}\right)^{1/2}, \quad \hat{\alpha}_{s2} = \left(\frac{v_2^2}{c_s^2} - 1\right)^{1/2} \tag{2.9}$$

and the subscript 2 denotes quantities associated with the crack tip velocity v_2. This singularity is always weaker than the conventional square-root singularity except at a single crack tip velocity $v_2 = \sqrt{2}c_s$ (at which $q_2 = \frac{1}{2}$). Accordingly, the stress intensity factor is defined by $k_{II} = \lim_{\xi \to 0^+} \sqrt{2\pi}\xi^{q_2}\sigma_{xy}\ (\xi, y = 0, t)$ for intersonic shear crack propagation. Following the same approach for the fundamental solution of intersonic crack propagation (Huang and Gao, 2001), we obtain the stress intensity factor around an intersonic shear crack tip with a moving dislocation at velocity w ($v_2 < w < c_l$) as

$$k_{II} = bk_0^{int}(t, w, v_2), \tag{2.10}$$

where

$$k_0^{\rm int}(t,w,v_2)$$
$$= -4\mu\sqrt{\frac{2}{\pi}}\,\alpha_{12}\hat{\alpha}_{s2}\,\frac{c_s^3(c_1-v_2)}{v_2^2(v_2^2-c_R^2)}\,f(v_2)\left(\frac{v_2^2-c_s^2}{c_1^2-v_2^2}\right)^{q_2}$$
$$\times\frac{w^2-c_R^2}{\sqrt{w+c_1\sqrt{w^2-c_s^2}\sqrt{w-v_2}}}\,\frac{s_{2-}(-1/(w-v_2))}{s_{2-}(0)}[(c_1-v_2)t]^{q_2-1}. \qquad (2.11)$$

Here functions $s_{2-}(\zeta)/s_{2-}(0)$ and $f(v_2)$, introduced by Huang and Gao (2001), are given by

$$\frac{s_{2-}(\zeta)}{s_{2-}(0)} = \exp\left\{-\frac{\zeta}{\pi}\int_{1/(c_1+v_2)}^{+\infty}\left[\frac{\pi}{2}+\left(\frac{\pi}{2}-\tan^{-1}V_-(r,v_2)\right)H^*(r,v_2)\right]\frac{{\rm d}r}{r(r-\zeta)}\right\}, \qquad (2.12)$$

$$f(v_2)$$
$$=\exp\left\{\left[\int_{1/(c_1+v_2)}^{1/(v_2+c_s)}-\int_{1/(v_2-c_s)}^{+\infty}\right]\tan^{-1}\left[\frac{4\alpha_{12}\hat{\alpha}_{s2}V_-(r,v_2)-(2-v_2^2/c_s^2)^2}{4\alpha_{12}\hat{\alpha}_{s2}+(2-v_2^2/c_s^2)^2V_-(r,v_2)}\right]\frac{{\rm d}r}{\pi r}\right\}, \qquad (2.13)$$

$$V_\pm(r,v)$$
$$=\frac{[2r^2-(v^2/c_s^2)(r\pm 1/v)^2]^2}{4r^2\sqrt{1-\frac{v^2}{c_1^2}}\sqrt{\frac{v^2}{c_s^2}-1}\sqrt{\left(r\mp\frac{1}{c_1-v}\right)\left(r\pm\frac{1}{c_1+v}\right)\left|r\pm\frac{1}{v-c_s}\right|\left|r\pm\frac{1}{v+c_s}\right|}}, \qquad (2.14)$$

$$H^*(r,v)=H\left(\frac{1}{v+c_s}-r\right)-H\left(r-\frac{1}{v-c_s}\right). \qquad (2.15)$$

Once again, the subscript 2 labels quantities associated with crack tip velocity v_2.

2.2. Crack-face sliding displacement in the fundamental solution

Huang and Gao (2001) obtained the fundamental solution for intersonic crack propagation, i.e., an initially stationary crack starting to propagate at a velocity v_1 ($v_1 > c_s$) once a pair of shear concentrated forces τ^* is applied at time $t=0$ to the initial crack tip $(x,y)=(0,0)$. The sliding displacement across crack faces in this fundamental solution depends on time t and coordinate x through their ratio x/t, i.e.,

$$\delta_1(x<v_1t,t)\equiv u_x(x<v_1t,y=0_+,t)-u_x(x<v_1t,y=0_-,t)=\delta_1\left(\frac{x}{t}\right) \qquad (2.16)$$

and δ_1 is given by (Huang and Gao, 2001, 2002)

$$\delta_1(w) = \frac{2}{\pi}\,\mathrm{PV}\int_{1/(c_1+v_1)}^{1/(v_1-w)} \mathrm{Im}[U_{1-}(\eta)]\,\mathrm{d}\eta, \tag{2.17}$$

where PV stands for the Cauchy principal value integral,

$$\begin{aligned}\mathrm{Im}[U_{1-}(\eta)] &= \frac{\sqrt{c_1 v_1}}{c_s^2}\frac{\tau^*}{\mu}\frac{s_{1+}(\eta)}{s_{1+}(1/v_1)}\frac{1-v_1\eta}{4\eta^2\sqrt{(c_1+v_1)\eta-1}}\\ &\times\left\{\frac{1}{1+V_-^2(\eta,v_1)}+\left[\frac{1}{1-V_-(\eta,v_1)}-\frac{1}{1+V_-^2(\eta,v_1)}\right]\right.\\ &\left.\times H\left(\eta-\frac{1}{v_1+c_s}\right)H\left(\frac{1}{v_1-c_s}-\eta\right)\right\}\end{aligned} \tag{2.18}$$

and

$$\frac{s_{1+}(\zeta)}{s_{1+}(1/v_1)} = \exp\left\{-\frac{\zeta-1/v_1}{\pi}\int_{1/(c_1-v_1)}^{+\infty}\frac{\tan^{-1}V_+(r,v_1)}{(r+1/v_1)(r+\zeta)}\,\mathrm{d}r\right\}. \tag{2.19}$$

The subscript 1 denotes quantities associated with the crack tip velocity v_1, and functions $V_\pm$ are given in Eq. (2.14). It can be verified that the function $\mathrm{Im}[U_{1-}(\eta)]$ in Eq. (2.18) is the same as that in Huang and Gao (2002).

2.3. Stress intensity factor around a decelerating crack tip

Similar to Freund (1972, 1990) and Huang and Gao (2002), the decelerating crack solution can be obtained from the fundamental solution for intersonic crack propagation (Huang and Gao, 2001) by negating the sliding displacement from the decelerating crack tip, $x = v_1 t^* + v_2(t - t^*)$, to the crack tip that would continue to propagate at an initial velocity v_1, $x = v_1 t$. This sliding displacement in the fundamental solution depends only on $w = x/t$, as shown in Eqs. (2.16) and (2.17). This implies that any given displacement level radiates out along the x-axis at a constant speed w. The range of w for negating the sliding displacement is from $(v_1 t^* + v_2(t - t^*))/t$ to v_1, corresponding to two crack tips discussed above. For a given velocity w within this range, the time t_w at which the displacement $\delta_1(w) = \delta_1(x/t)$ arrives at the decelerated crack tip location $x = v_1 t^* + v_2(t - t^*)$ is given by

$$t_w = \frac{v_1 - v_2}{w - v_2}\,t^*. \tag{2.20}$$

For a dislocation moving with velocity w and (half) Burgers vector b after being emitted from a shear crack tip at time $t = 0$, the stress intensity factor of the crack tip propagating with velocity v_2 ($v_2 < w$) is $bk_0(t, w, v_2)$, where $k_0(t, w, v_2)$ is given in Eqs. (2.6) and (2.11) for sub-Rayleigh ($v_2 < c_R$) and intersonic ($v_2 > c_s$) crack propagation, respectively. If a dislocation with the Burgers vector $\mathrm{d}\delta_1$ begins moving at time $t=t_w$ (instead of $t=0$), then the stress intensity factor is $k_0(t-t_w,w,v_2)\,\mathrm{d}\delta_1/2$. Since both δ_1 in Eq. (2.17) and t_w in Eq. (2.20) are functions of w, the stress intensity factor of the decelerated crack tip can be summed over the entire range of w, from $(v_1t^*+v_2(t-t^*))/t$ to v_1. Since stresses in the fundamental solution are not singular around the decelerated crack tip $x=v_1t^*+v_2(t-t^*)$, only the moving dislocation solution contributes to the stress intensity factor K_{II} of the decelerated crack tip, which gives K_{II} as

$$K_{\mathrm{II}}(t,v_1,v_2)=-\int_{v_1}^{(v_1t^*+v_2(t-t^*))/t} k_0(t-t_w,w,v_2)\,\frac{1}{2}\,\frac{\mathrm{d}\delta_1}{\mathrm{d}w}\,\mathrm{d}w, \tag{2.21}$$

where

$$\frac{\mathrm{d}\delta_1}{\mathrm{d}w}=\frac{2}{\pi}\,\frac{1}{(v_1-w)^2}\,\mathrm{Im}\left[U_{1-}\left(\frac{1}{v_1-w}\right)\right] \tag{2.22}$$

is obtained from Eq. (2.17).

2.3.1. Deceleration to a sub-Rayleigh crack tip velocity

For an intersonic crack tip velocity v_1 decelerating to a sub-Rayleigh one v_2, the substitution of k_0^{sub} in Eq. (2.6) into Eq. (2.21) and the change of integration variable to $\eta=1/(v_1-w)$ gives the stress intensity factor of the decelerated sub-Rayleigh crack tip as

$$\begin{aligned}K_{\mathrm{II}}(t,v_1,v_2<c_{\mathrm{R}})=&-\sqrt{\frac{2(c_{\mathrm{s}}+v_2)}{\pi}}\,\frac{\mu}{\pi(c_{\mathrm{R}}+v_2)}\,\frac{\kappa_2c_{\mathrm{s}}^2}{\alpha_{\mathrm{s}2}v_2^2}\\&\times\mathrm{PV}\int_{\frac{1}{(v_1-v_2)(1-t^*/t)}}^{+\infty}\frac{(v_1+c_{\mathrm{R}})\eta-1}{\sqrt{(v_1-v_2)(t-t^*)\eta-t}\sqrt{(v_1+c_{\mathrm{s}})\eta-1}}\\&s_-^{\mathrm{sub}}\left[-\frac{\eta}{(v_1-v_2)\eta-1}\right]\mathrm{Im}[U_{1-}(\eta)]\,\mathrm{d}\eta,\end{aligned} \tag{2.23}$$

where s_-^{sub} and $\mathrm{Im}[U_{1-}]$ are given in Eqs. (2.7) and (2.18), respectively. It is straightforward to show that Eq. (2.23) degenerates to the stress intensity factor obtained by Huang and Gao (2002) when $v_2=0$ (i.e., a suddenly arrested intersonic crack). After lengthy calculations, it can be shown that Eq. (2.23) has the asymptotic limit of a sub-Rayleigh crack as time $t\to\infty$,

$$K_{\mathrm{II}}(t\to\infty)\sim\tau^*\sqrt{\frac{2}{\pi[v_1t^*+v_2(t-t^*)]}}\,\frac{1-v_2/c_{\mathrm{R}}}{s_+^{\mathrm{sub}}(1/v_2)\sqrt{1-v_2/c_{\mathrm{s}}}}, \tag{2.24}$$

where its right-hand side is exactly the stress intensity factor for a crack tip propagating with velocity v_2 ($<c_{\mathrm{R}}$) and subjected to a pair of shear forces τ^* at a distance $v_1t^*+v_2(t-t^*)$ behind the crack tip. The expression for s_+^{sub} is given in Eq. (2.7).

Eq. (2.23) gives a vanishing stress intensity factor at the instant of crack tip deceleration, $K_{\mathrm{II}}(t\to t^*+0)=0$. As the stress singularity suddenly increases from a weaker

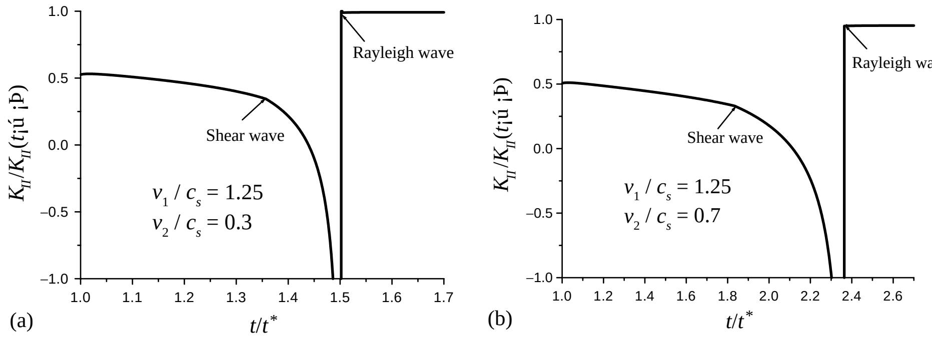

Fig. 1. The stress intensity factor, $K_{\rm II}$, normalized by $K_{\rm II}(t \to \infty)$, around a crack tip that suddenly decelerates from an intersonic velocity, v_1, to a sub-Rayleigh velocity, v_2. Time t is normalized by the crack propagation time t^*; Poisson's ratio $v = \frac{1}{3}$. The initial crack tip velocity is $v_1 = 1.25c_{\rm s}$, and is decelerated to (a) $v_2 = 0.3c_{\rm s}$ and (b) $v_2 = 0.7c_{\rm s}$.

singularity for an intersonic crack to the square-root one ($r^{-1/2}$) for a sub-Rayleigh crack, the corresponding stress intensity factor vanishes at this instant. This implies that, contrary to the sub-Rayleigh crack growth, an intersonic crack decelerating to a sub-Rayleigh one displays history dependence. The stress intensity factor of the decelerated, sub-Rayleigh crack tip does not reach its limit in Eq. (2.24) instantaneously. In fact, this is reasonable because both shear and Rayleigh waves are still trailing behind the crack tip as its velocity suddenly drops to v_2 ($< c_{\rm R}$) at the instant $t = t^*$. The time for Rayleigh wave to catch the decelerated, sub-Rayleigh crack tip is $t = (v_1 - v_2)/(c_{\rm R} - v_2)t^*$. It will be interesting to examine the stress intensity factor after all the waves trailing behind have passed the decelerated, sub-Rayleigh crack tip, i.e., $t > (v_1 - v_2)/(c_{\rm R} - v_2)t^*$.

Fig. 1 shows the stress intensity factor $K_{\rm II}$ in Eq. (2.23) versus normalized time t/t^* for the initial intersonic crack tip velocity $v_1 = 1.25c_{\rm s}$ and decelerated sub-Rayleigh crack tip velocities $v_2 = 0.3c_{\rm s}$ and $0.7c_{\rm s}$, where $K_{\rm II}$ is normalized by its limit $K_{\rm II}(t \to \infty)$ in the right-hand side of Eq. (2.24). The Poisson's ratio is $v = \frac{1}{3}$, which gives a longitudinal wave speed of $c_{\rm l}=2c_{\rm s}$ and a Rayleigh wave speed of $c_{\rm R}=0.93c_{\rm s}$. The stress intensity factor $K_{\rm II}$ clearly differs from the limit $K_{\rm II}(t \to \infty)$. However, immediately after the Rayleigh wave arrives ($t \to (v_1 - v_2)/(c_{\rm R} - v_2)t^* + 0$), $K_{\rm II}$ reaches $K_{\rm II}(t \to \infty)$ instantaneously. Therefore, after an intersonic crack decelerates to a sub-Rayleigh one, the limit of sub-Rayleigh stress intensity factor in the right-hand side of Eq. (2.24) is reached after a finite delay for all waves trailing behind to pass the decelerated crack tip.

2.3.2. Deceleration to another intersonic crack tip velocity

For an intersonic crack tip velocity v_1 decelerating to another intersonic one v_2, the substitution of $k_0^{\rm int}$ in Eq. (2.11) into Eq. (2.21) and the change of integration variable to $\eta = 1/(v_1 - w)$ give the stress intensity factor of the decelerated intersonic crack tip

$$K_{II}(t, v_1, v_2 > c_s)$$
$$= -\sqrt{\frac{2}{\pi}}\frac{4\mu\alpha_{12}\hat{\alpha}_{s2}}{\pi}\frac{c_s^3(c_1-v_2)}{v_2^2(v_2^2-c_R^2)}f(v_2)\left(\frac{v_2^2-c_s^2}{c_1^2-v_2^2}\right)^{q_2}$$
$$\times \mathrm{PV}\int_{\frac{1}{(v_1-v_2)(1-t^*/t)}}^{+\infty}\frac{[(v_1+c_R)\eta-1][(v_1-c_R)\eta-1]}{\sqrt{(c_1+v_1)\eta-1}\sqrt{(v_1+c_s)\eta-1}\sqrt{(v_1-c_s)\eta-1}\sqrt{(v_1-v_2)\eta-1}}$$
$$\times\frac{s_{2-}[-\eta/((v_1-v_2)\eta-1)]}{s_{2-}(0)}\left\{(c_1-v_2)\left[t-\frac{(v_1-v_2)t^*\eta}{(v_1-v_2)\eta-1}\right]\right\}^{q_2-1}$$
$$\times \mathrm{Im}[U_{1-}(\eta)]\,\mathrm{d}\eta, \tag{2.25}$$

where $s_{2-}/s_{2-}(0)$ and $\mathrm{Im}[U_{1-}]$ are given in Eqs. (2.12) and (2.18), respectively. After some lengthy calculations, it can be shown that Eq. (2.25) agrees with the fundamental solution for intersonic crack propagation (Huang and Gao, 2001) at the limit $v_2 = v_1$ (i.e., no change in crack tip velocity). It can also be shown that, as time $t \to \infty$, Eq. (2.25) has the asymptotic limit of an intersonic crack propagating with velocity v_2:

$$K_{II}(t\to\infty) \sim 4\tau^*\sqrt{\frac{2}{\pi}}\frac{\alpha_{12}\hat{\alpha}_{s2}c_s^4}{\sqrt{c_1v_2^3(v_2^2-c_R^2)}}f(v_2)\left[\frac{v_2^2-c_s^2}{(c_1+v_2)v_2}\right]^{q_2}$$
$$\frac{s_{2+}(0)}{s_{2+}(1/v_2)}[v_1t^* + v_2(t-t^*)]^{q_2-1}, \tag{2.26}$$

where its right-hand side is exactly the stress intensity factor in the fundamental solution for intersonic crack propagation (Huang and Gao, 2001) except that the pair of point forces is at a distance $v_1t^* + v_2(t-t^*)$ behind the crack tip; $f(v_2)$ and $s_{2+}/s_{2+}(1/v_2)$ are given in Eqs. (2.13) and (2.19), respectively.

The stress singularities q_1 and q_2 around an intersonic crack tip before and after deceleration are given by

$$q_i = \frac{1}{\pi}\tan^{-1}\left[4\sqrt{1-\frac{v_i^2}{c_1^2}}\sqrt{\frac{v_i^2}{c_s^2}-1}\bigg/\left(2-\frac{v_i^2}{c_s^2}\right)^2\right] \quad (i=1,2)$$

for corresponding crack tip velocities v_1 and v_2 [see Eqs. (2.8) and (2.9)]. For $q_2 > q_1$ (stronger singularity after deceleration), Eq. (2.25) gives a vanishing stress intensity factor immediately after crack tip deceleration, $K_{II}(t \to t^* + 0) = 0$. For $q_2 < q_1$ (weaker singularity after deceleration), Eq. (2.25) approaches infinity at the same instant, $K_{II}(t \to t^* + 0) \to \infty$. For $q_2 = q_1$ (same singularity), K_{II} remains finite after crack tip deceleration. This implies once again that, contrary to sub-Rayleigh crack propagation, the stress intensity factor around an intersonic crack tip depends not only on the instantaneous crack tip velocity but also on the history of crack propagation because both shear and Rayleigh waves are always trailing behind the intersonic crack tip.

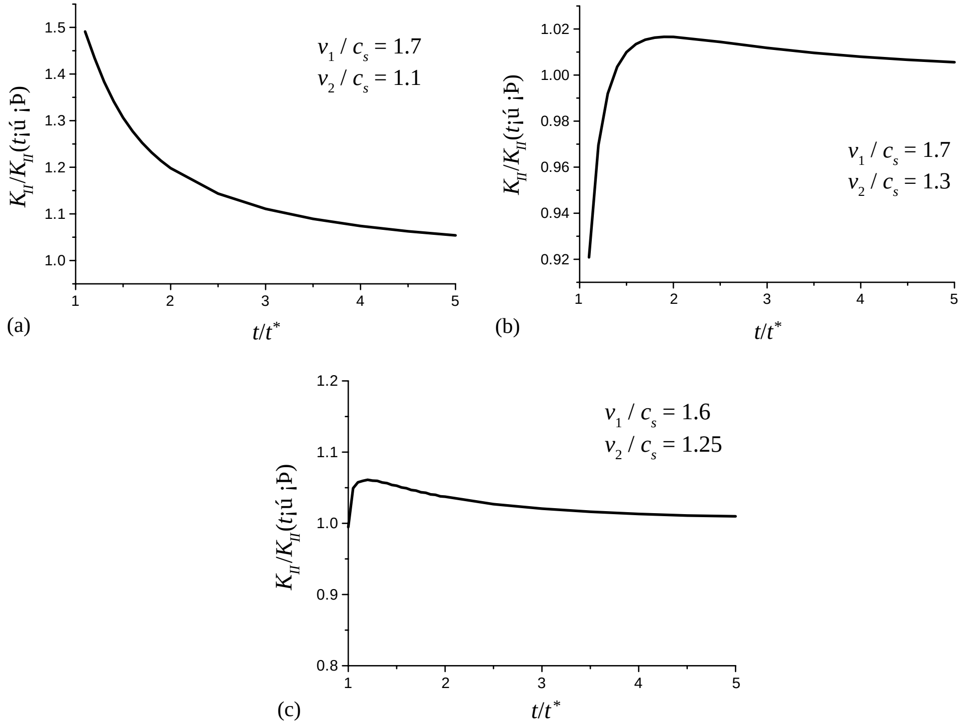

Fig. 2. The stress intensity factor, K_{II}, normalized by $K_{\mathrm{II}}(t \to \infty)$, around a crack tip that suddenly decelerates from an intersonic velocity, v_1, to a another intersonic velocity, v_2. Time t is normalized by the crack propagation time t^*; Poisson's ratio $\nu = \frac{1}{3}$. The crack tip velocities are (a) $v_1 = 1.7c_s$, $v_2 = 1.1c_s$; (b) $v_1 = 1.7c_s$, $v_2 = 1.3c_s$; (c) $v_1 = 1.6c_s$, $v_2 = 1.25c_s$.

Fig. 2 shows the stress intensity factor K_{II} in Eq. (2.25) versus normalized time t/t^* for initial intersonic crack tip velocity $v_1 = 1.7c_s$ and decelerated crack tip velocity $v_2 = 1.1c_s$ and $1.3c_s$, where K_{II} is normalized by its limit $K_{\mathrm{II}}(t \to \infty)$ in the right-hand side of Eq. (2.26). The Poisson's ratio is $\nu = \frac{1}{3}$, which gives a longitudinal wave speed of $c_l = 2c_s$ and a Rayleigh wave speed of $c_R = 0.93c_s$. For $v_1 = 1.7c_s$ and $v_2 = 1.1c_s$, the decelerated crack tip has a weaker singularity and the stress intensity factor bursts immediately after crack deceleration. In contrast, the decelerated crack tip has a stronger singularity for $v_1 = 1.7c_s$ and $v_2 = 1.3c_s$, and the stress intensity factor vanishes at the instant of crack deceleration. For the case of $v_1 = 1.6c_s$ and $v_2 = 1.25c_s$ presented in Fig. 1c, the decelerated crack tip has an almost identical singularity as the previous one. Consequently, the stress intensity factor remains finite after crack deceleration and only varies slightly during the deceleration history. That suggests sudden deceleration is likely to occur when the deceleration causes little distortion of the crack tip singularity or the energy flux into the crack tip. It is also observed that the limit $K_{\mathrm{II}}(t \to \infty)$ is only reached asymptotically, i.e., the limiting K_{II} is not at finite time since the shear and Rayleigh waves trailing behind can never catch up with the intersonic crack tip.

3. Accelerating crack

Consider the same problem as in Section 2 except that the crack tip velocity increases after a finite propagation. At time $t=0$, a pair of concentrated shear forces of magnitude τ^* is imposed at the crack tip, which drives the crack to extend along the positive x-axis at a velocity v_1. At a later time $t=t^*$, the crack suddenly accelerates to a higher intersonic velocity v_2 ($> v_1$) after an extension of $l = v_1 t^*$. Similar to Section 2, this problem can also be viewed as the superposition of following two sub-problems.

a. Continuous crack propagation at the initial crack tip velocity v_1. When $v_1 > c_s$, this is the fundamental solution obtained by Huang and Gao (2001). While if $v_1 < c_s$, it is the subsonic fundamental solution which was studied by Freund (1972). Although he only gave the stress intensity factor for a mode I crack, one can get the stress of a mode II crack in the same manner. The shear stress ahead of the crack tip ($x > 0, y=0$) in this fundamental solution has the form $\sigma_{xy}^{\mathrm{F}}(x, y=0, t)=t^{-1}\sigma_1(x/t)=t^{-1}\sigma_1(w)$, where $w=x/t$, and the subscript 1 is associated with the crack tip velocity v_1.
b. Negation of the above shear stress in the fundamental solution between the actual crack tip, $x=v_1t^*+v_2(t-t^*)$, and the fictitious crack tip would continue to propagate at initial velocity v_1, $x=v_1t$. This is achieved by imposing a series of moving point forces in the appropriate sequence. This is because the above shear stress to be negated can be written as

$$\sigma_{xy}^{\mathrm{F}}(x, y=0, t) = \int_{v_1 t}^{v_1 t^* + v_2(t-t^*)} \delta(x - x')\frac{1}{t}\,\sigma_1\left(\frac{x'}{t}\right)\mathrm{d}x'$$

$$= \int_{v_1}^{(v_1 t^* + v_2(t-t^*))/t} \delta(x - wt)\sigma_1(w)\,\mathrm{d}w,$$

which can be viewed as a series of point forces $\sigma_1(w)\,\mathrm{d}w$ moving with velocity w, where δ is the Dirac delta function.

3.1. A moving point force on crack faces of a propagating crack tip

An infinite solid containing a semi-infinite crack on the negative x-axis is stress free and at rest everywhere for time $t < 0$. A pair of unit shear point forces is applied to the crack tip at time $t=0$, which drives the crack tip to propagate along the positive x-axis at an intersonic velocity v_2. The unit point forces also move in the same direction at a lower velocity w ($< v_2$) on crack faces.

The analysis is identical to the fundamental solution of intersonic crack propagation (Huang and Gao, 2001) except that the unit point forces are moving. The boundary conditions become

$$\sigma_{yy}(x, y=0, t) = 0, \qquad \sigma_{xy}(x < v_2 t, y=0, t) = -\delta(x - wt)H(t),$$
$$u_x(x > v_2 t, y=0, t) = 0. \tag{3.1}$$

Only the stress intensity factor, which is pertinent to the accelerating crack problem, is given in the following. The stress intensity factor around an intersonic crack tip is defined by $k_{\mathrm{II}} = \lim_{\xi\to 0^+} \sqrt{2\pi}\xi^{q_2}\sigma_{xy}\ (\xi, y = 0, t)$, where the stress singularity q_2 is given in Eq. (2.8). Following the same approach of Huang and Gao (2001), we have obtained the stress intensity factor around an intersonic shear crack tip subjected to a pair of unit point forces moving at a lower velocity w ($w < v_2$) on crack faces

$$k_{\mathrm{II}} = k(t, w, v_2), \tag{3.2}$$

where

$$k(t, w, v_2) = 4\sqrt{\frac{2}{\pi}}\alpha_{l2}\hat{\alpha}_{s2}\frac{c_s^4(c_l - v_2)}{v_2^2(v_2^2 - c_R^2)} f(v_2)\left(\frac{v_2^2 - c_s^2}{c_l^2 - v_2^2}\right)^{q_2} \times \frac{1}{\sqrt{c_l - w}\sqrt{v_2 - w}} \frac{s_{2+}(0)}{s_{2+}(1/(v_2 - w))}[(c_l - v_2)t]^{q_2 - 1}. \tag{3.3}$$

Here α_{l2}, $\hat{\alpha}_{s2}$ and $f(v_2)$ are given in Eqs. (2.9) and (2.13), respectively, $s_{2+}(\zeta)/s_{2+}(0)$ is given by

$$\frac{s_{2+}(\zeta)}{s_{2+}(0)} = \exp\left\{-\frac{\zeta}{\pi}\int_{1/(c_l - v_2)}^{+\infty} \frac{\tan^{-1} V_+(r, v_2)}{r(r + \zeta)}\,\mathrm{d}r\right\}, \tag{3.4}$$

which is consistent with Eq. (2.19), and V_+ is given in Eq. (2.14).

3.2. Shear stress ahead of the crack tip in the fundamental solution

Freund (1972), and Huang and Gao (2001) obtained the fundamental solution for sub-Rayleigh crack propagation and intersonic crack propagation respectively, i.e., an initially stationary crack starting to propagate at a velocity v_1 ($v_1 < c_R$ or $v_1 > c_s$, respectively) once a pair of shear concentrated forces τ^* are applied at time $t = 0$ to the initial crack tip $(x, y) = (0, 0)$. Although Freund only gave the stress intensity factor of mode I crack, one can get the stress of mode II crack in the same way.

3.2.1. Shear stress ahead of the crack tip in the intersonic fundamental solution

The shear stress ahead of the intersonic crack tip in this fundamental solution is given by

$$\sigma_{xy}(x > v_1 t, y = 0, t) = \frac{1}{t}\,\sigma_1^{\mathrm{int}}\left(\frac{x}{t}\right) \tag{3.5}$$

and

$$\sigma_1^{\mathrm{int}}(w) = \frac{4}{\pi}\sqrt{\frac{v_1}{c_l^3}}\,\tau^*\,\frac{s_{1+}(0)}{s_{1+}(1/v_1)}\,\frac{c_s^3(c_l + w)\sqrt{c_l - w}\sqrt{w^2 - c_s^2}}{w^3(w^2 - c_R^2)\sqrt{w - v_1}} \times \frac{s_{1-}(0)}{s_{1-}(-1/(w - v_1))}, \tag{3.6}$$

where $s_{1+}(0)/s_{1+}(1/v_1)$ is given in Eq. (2.19),

$$\frac{s_{1-}(\zeta)}{s_{1-}(0)} = \exp\left\{-\frac{\zeta}{\pi}\int_{1/(c_1+v_1)}^{+\infty}\left[\frac{\pi}{2}+\left(\frac{\pi}{2}-\tan^{-1}V_-(r,v_1)\right)H^*(r,v_1)\right]\frac{\mathrm{d}r}{r(r-\zeta)}\right\}, \quad (3.7)$$

functions V_- and H^* are given in Eqs. (2.14) and (2.15), respectively, and once again, the subscript 1 labels quantities associated with the crack tip velocity v_1.

3.2.2. Shear stress ahead of the crack tip in the subsonic fundamental solution

The shear stress ahead of the sub-Rayleigh crack tip in this fundamental solution is given by

$$\sigma_{xy}(x > v_1 t, y = 0, t) = \frac{1}{t}\,\sigma_1^{\mathrm{sub}}\left(\frac{x}{t}\right) \quad (3.8)$$

and

$$\sigma_1^{\mathrm{sub}}(w) = \frac{\tau^*}{\pi w^3 c_{\mathrm{R}}\sqrt{w-v_1}}\,\frac{\sqrt{v_1 c_{\mathrm{s}}}}{\kappa s_+^{\mathrm{sub}}(1/v_1)s_-^{\mathrm{sub}}(-1/(w-v_1))}\,\frac{(c_{\mathrm{R}}^2-v_1^2)v_1^2}{(w+c_{\mathrm{R}})}\,\frac{1}{\sqrt{|w-c_{\mathrm{s}}|}}$$
$$\times\left[\frac{4}{c_1 c_{\mathrm{s}}}\sqrt{c_1^2-w^2}\sqrt{|w^2-c_{\mathrm{s}}^2|}-\left(2-\frac{w^2}{c_{\mathrm{s}}^2}\right)^2 H(c_{\mathrm{s}}-w)\right]H(c_1-w), \quad (3.9)$$

where

$$\kappa = 4\alpha_{11}\alpha_{\mathrm{s}1}-(1+\alpha_{\mathrm{s}1}^2)^2, \quad \alpha_{11} = \left(1-\frac{v_1^2}{c_1^2}\right)^{1/2}, \quad \alpha_{\mathrm{s}1} = \left(1-\frac{v_1^2}{c_{\mathrm{s}}^2}\right)^{1/2}$$

$$s_\pm^{\mathrm{sub}}(\zeta) = \exp\left\{-\frac{1}{\pi}\int_{1/(c_1\mp v_1)}^{1/(c_{\mathrm{s}}\mp v_1)}\tan^{-1}\left[\frac{4c_{\mathrm{s}}^3 r^2\sqrt{(1-v_1 r)^2-c_{\mathrm{s}}^2 r^2}\sqrt{c_1^2 r^2-(1-v_1 r)^2}}{c_1[2c_{\mathrm{s}}^2 r^2-(1\pm v_1 r)^2]^2}\right]\frac{\mathrm{d}r}{r\pm\zeta}\right\}. \quad (3.10)$$

It is worthwhile to note that the stress decreases gradually with the increasing of w, changes its sign at $w = c_{\mathrm{R}}$, tends to negative infinity at $w = c_{\mathrm{s}}$, then jumps back to a positive value when w exceeds c_{s}.

3.3. Stress intensity factor around an accelerating crack tip

The accelerating crack solution can be obtained from the fundamental solution for intersonic crack propagation (Huang and Gao, 2001) by negating the shear stress from the accelerating crack tip, $x = v_1 t^* + v_2(t - t^*)$, to the crack tip that would continue to propagate at initial velocity v_1, $x = v_1 t$. As discussed before, this shear stress in the fundamental solution has the form $t^{-1}\sigma_1(x/t)$, which can be equivalently represented by a series of point forces $\sigma_1(w)\,\mathrm{d}w$ moving with velocity w, where $w=x/t$. The range of w for negating the shear stress is from v_1 to $(v_1t^*+v_2(t-t^*))/t$, corresponding to two crack tips discussed above. For a given velocity w in this range, the time t_w at which the point force $\sigma_1(w)\,\mathrm{d}w$ arrives at the accelerated crack tip location $x=v_1t^*+v_2(t-t^*)$ is also given by Eq. (2.20).

For a pair of unit shear point forces moving with velocity w on crack faces, the stress intensity factor of the crack tip propagating with an intersonic velocity v_2 ($v_2>w$) is $k(t,w,v_2)$, where $k(t,w,v_2)$ is given in Eq. (3.3). If a pair of shear point forces $\sigma_1(w)\,\mathrm{d}w$ begins moving at time $t=t_w$ (instead of $t=0$), then the stress intensity factor is $k(t-t_w,w,v_2)\sigma_1(w)\,\mathrm{d}w$. The values of σ_1 are given in Eqs. (3.6) and (3.9) for intersonic and subsonic crack propagation, respectively. Since both σ_1 in Eqs. (3.6) and (3.9) and t_w in Eq. (2.20) are functions of w, the stress intensity factor of the accelerated crack tip can be summed over the entire range of w, from v_1 to $(v_1t^*+v_2(t-t^*))/t$. Since stresses in the fundamental solution are not singular around the accelerated crack tip $x=v_1t^*+v_2(t-t^*)$, only the moving point-force solution contributes to the stress intensity factor K_{II} of the accelerated crack tip, which gives K_{II} as

$$K_{\mathrm{II}}(t,v_1,v_2)=\int_{(v_1t^*+v_2(t-t^*))/t}^{v_1} k(t-t_w,w,v_2)\sigma_1(w)\,\mathrm{d}w. \tag{3.11}$$

The substitution of k, σ_1 and t_w in Eqs. (3.3), (3.6), (3.9) and (2.20) into Eq. (3.11) and the change of integration variable to $\eta=1/(w-v_1)$ give the stress intensity factor presiding over an accelerated intersonic or sub-Rayleigh crack tip as

$$\begin{aligned}K_{\mathrm{II}}(t,v_1,v_2)=&\sqrt{\frac{2}{\pi}}\,4\alpha_{l2}\hat{\alpha}_{s2}\,\frac{c_s^4(c_l-v_2)}{v_2^2(v_2^2-c_R^2)}\,f(v_2)\left(\frac{v_2^2-c_s^2}{c_l^2-v_2^2}\right)^{q_2}\\ &\times\int_{\frac{1}{(v_2-v_1)(1-t^*/t)}}^{+\infty}\frac{1}{\sqrt{(c_l-v_1)\eta-1}\sqrt{(v_2-v_1)\eta-1}}\,\frac{s_{2+}(0)}{s_{2+}[\frac{\eta}{(v_2-v_1)\eta-1}]}\\ &\left\{(c_l-v_2)\left[t-\frac{(v_2-v_1)t^*\eta}{(v_2-v_1)\eta-1}\right]\right\}^{q_2-1}\sigma_1\left(v_1+\frac{1}{\eta}\right)\frac{\mathrm{d}\eta}{\eta},\end{aligned} \tag{3.12}$$

where functions f and $s_{2+}/s_{2+}(0)$ are given in Eqs. (2.13) and (3.4), and the values of σ_1 are assigned in Eqs. (3.6) and (3.9) for the intersonic and sub-Rayleigh cases, respectively. After some lengthy calculations, it can be shown that Eq. (3.12) agrees with the fundamental solution for intersonic crack propagation (Huang and Gao, 2001) at the limit $v_2=v_1$, and also with the asymptotic limit in Eq. (2.26) as time $t\to\infty$.

The stress intensity factor in Eq. (3.11) at the instant of crack tip acceleration ($t\to t^*+0$) is zero, finite, and infinite depending on whether the stress singularity of accelerated crack tip is stronger than ($q_2>q_1$), the same as ($q_2=q_1$), or weaker than ($q_2<q_1$) that before acceleration. The same asymptotic behavior has also been observed in Section 2 for a decelerated crack tip. When v_1 is sub-Rayleigh, the square-root singularity ($q_1=\frac{1}{2}$) of stress always prevails. Accordingly, the stress intensity factor at the instant of crack tip acceleration from a sub-Rayleigh to an intersonic speed always vanishes

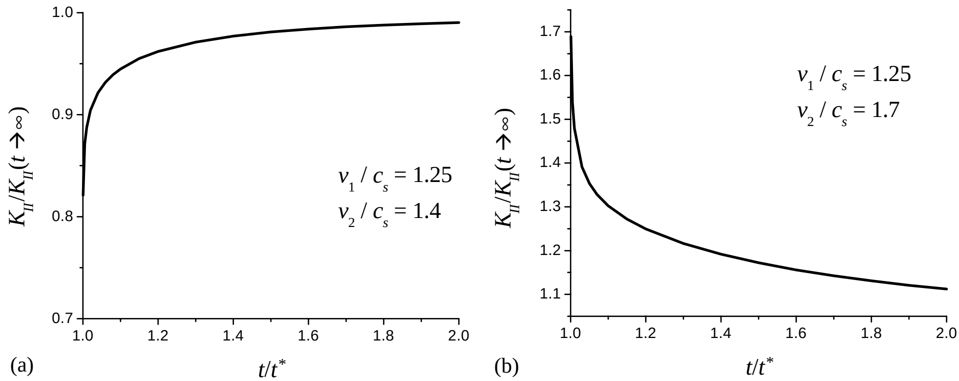

Fig. 3. The stress intensity factor, K_{II}, normalized by $K_{\mathrm{II}}(t \to \infty)$, around a crack tip that suddenly accelerates from an intersonic velocity, v_1, to another intersonic velocity, v_2. Time t is normalized by the crack propagation time t^*; Poisson's ratio $\nu = \frac{1}{3}$. The initial crack tip velocity is $v_1 = 1.25c_{\mathrm{s}}$, the crack is suddenly accelerated to (a) $v_2 = 1.4c_{\mathrm{s}}$ and (b) $v_2 = 1.7c_{\mathrm{s}}$.

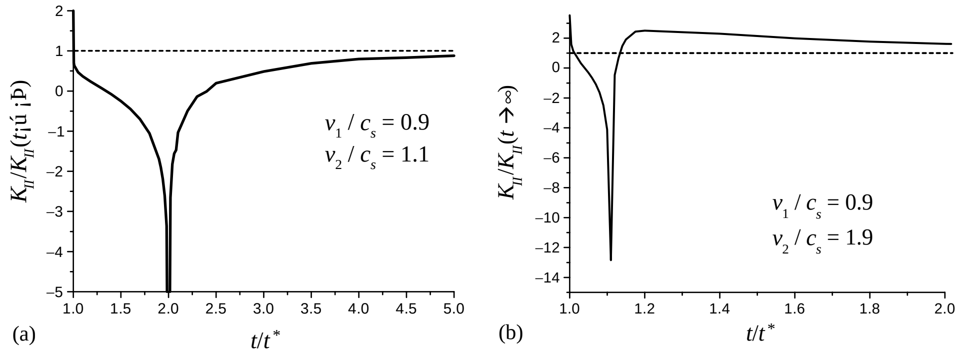

Fig. 4. The stress intensity factor, K_{II}, normalized by $K_{\mathrm{II}}(t \to \infty)$, around a crack tip that suddenly accelerates from a sub-Rayleigh velocity, v_1, to an intersonic velocity, v_2. Time t is normalized by the crack propagation time t^*; Poisson's ratio $\nu = \frac{1}{3}$. The initial velocity is $v_1 = 0.9c_{\mathrm{s}}$, and is accelerated to (a) $v_2 = 1.1c_{\mathrm{s}}$ and (b) $v_2 = 1.9c_{\mathrm{s}}$.

except for the case of $v_2 = \sqrt{2}c_{\mathrm{s}}$. This implies once again that, contrary to sub-Rayleigh crack growth, intersonic crack propagation has the history dependence.

Figs. 3 and 4 show the stress intensity factor K_{II} in Eq. (3.12) versus normalized time t/t^*, where K_{II} is normalized by its limit $K_{\mathrm{II}}(t \to \infty)$ in the right-hand side of Eq. (2.26). The Poisson's ratio is $\nu = \frac{1}{3}$, which gives a longitudinal wave speed of $c_{\mathrm{l}} = 2c_{\mathrm{s}}$ and a Rayleigh wave speed of $c_{\mathrm{R}} = 0.93c_{\mathrm{s}}$. Fig. 3 refers to the case of crack acceleration from an intersonic speed $v_1 = 1.25c_{\mathrm{s}}$ to a different intersonic speed. Two calculations are carried out for $v_2 = 1.4c_{\mathrm{s}}$ and $v_2 = 1.7c_{\mathrm{s}}$. For the first case (Fig. 3a), the crack tip singularity increases after the velocity jump, so the corresponding stress intensity factor rises, first rapidly and then gradually, from zero to unity. For the second case

(Fig. 3b), the crack tip singularity decreases after the velocity jump, the corresponding stress intensity factor declines, first rapidly and then gradually, from infinity to unity. After sudden velocity jump, the steady-state intersonic solution corresponding to v_2 cannot be fully achieved in a finite time period.

When a crack travels at a subsonic speed, the shear wave front induced by the sudden application of a pair of tangential point forces travels ahead of the crack tip. If that (previously subsonic) crack suddenly accelerates to an intersonic cracking speed, one would expect distinct transitions at two critical instants. The first instant happens right at the velocity switch, a sudden drop of the stress singularity (from $-\frac{1}{2}$ to $-q$), leading to a rapid declination of stress. The second critical instant occurs when the intersonic crack catches up with the shear wave front generated by the point forces. The subsonic stress solution, as described in Eq. (3.9), includes a jump in the integrand at shear wave speed, and consequently a sudden slop change in the shear stress response. Moreover, due to the singularity in Eq. (3.9), the stress intensity factor will decrease to negative infinity when the crack catches up with the shear wave. To visualize the anticipated changes in the crack tip stress intensity factor, we plot in Fig. 4 the stress intensity factor history of a sub-Rayleigh crack with an initial tip velocity $v_1=0.9c_s$ accelerating suddenly to the crack tip velocity of $v_2=1.1c_s$ (Fig. 4a) and $1.9c_s$ (Fig. 4b). The former case corresponds to the cross-over of the velocity forbidden zone from the Rayleigh wave speed to the shear wave speed; while the latter simulates the velocity jump from just under the Rayleigh wave speed to just under the longitudinal wave speed. For both cases, the crack tip in the previous sub-Rayleigh regime has a stronger singularity ($r^{-1/2}$), so the stress intensity factors are unbounded immediately after the crack acceleration. The second critical instances for two cases are $t = 2t^*$ when $v_2 = 1.1c_s$, and $t = (10/9)t^*$ when $v_2 = 1.9c_s$, as evidenced in Figs. 4a and b, respectively. One observes from Figs. 4a and b that the sign of shear stress changes after the velocity jump. The crack tip is first sheared in one direction, then sheared in the opposite direction, due to the intersonic crack catches up with, then breaks away from the shear wave front generated by the point forces. In contrast to a mode I crack, a mode II crack can be driven to propagate by either positive or negative shear stress. Therefore, the sudden velocity change of a shear crack confronts no insurmountable barrier. That observation may shed some light to the cross-over of the velocity forbidden zone between the Rayleigh wave and the shear wave speed. It is also observed that the limit $K_{\text{II}}(t \to \infty)$ is only reached asymptotically, i.e., not at any finite time since the shear and Rayleigh waves trailing behind need a long time to fade away.

4. Concluding remarks and discussion

Motivated by the recent progress on intersonic crack growth that undergoing abrupt changes in crack tip velocity, the present work presents analytical solutions for a sub-Rayleigh or an intersonic crack either accelerating or decelerating to a different (sub-Rayleigh or intersonic) cracking speed. Four cases are examined. The case of crack acceleration from a sub-Rayleigh speed to an intersonic speed is explored to delineate two situations: (1) crack speeding across the forbidden velocity zone from the Rayleigh wave speed to the shear wave speed, as shown in Fig. 4a; and (2) the observed phenomenon of crack speed jump from just below the Rayleigh wave speed to just below the longitudinal wave speed (Rosakis et al., 1999; Needleman, 1999; Needleman and Rosakis, 1999; Abraham and Gao, 2000; Gao et al., 2001), as shown in Fig. 4b. The case of crack deceleration from an intersonic speed to the sub-Rayleigh regime features the catch up of the trailing waves, namely the sub-Rayleigh solution is recovered when the trailing shear wave and Rayleigh surface wave catch up with the decelerated crack tip. The cases of an intersonic crack deceleration or acceleration to another intersonic crack are shown to be extensively history dependent. The effect of previous intersonic crack propagation history cannot be erased over a finite time span. This is the fundamental difference between sub-Rayleigh and intersonic cracks.

For the subsonic case, the method of superposition can be exercised at any time. For a crack propagating at a subsonic speed, the stress intensity factor does not depend on the history if the crack is loaded by time-independent force (Freund 1972, 1990). Regardless of the variation in crack speeds, the elastic field emitted from the crack tip after the velocity change is always self-similar. Unfortunately, the recent works by Huang and Gao (2002), as well as our calculations, show the strong history dependence of the stress intensity factor for an intersonic crack.

A propagating crack differs from a moving dislocation since the latter possesses inertia. Accordingly, an infinite amount of time is required for a moving dislocation, subsonic or intersonic, to approach equilibrium. In this aspect, a sub-Rayleigh crack is very different from a subsonic dislocation. However, in the intersonic regime, cracks and dislocations show similar behavior in that they require time to change from one steady state to another.

For a crack to change its speed continuously, Broberg (1999b) argued that the method of superposition might be used repeatedly. One then might expect that the present solution could serve as a building block for the general crack propagating history in the intersonic or sub-Rayleigh ranges. However, a distinction has to be made between the solutions of the first and the subsequent velocity jumps. When the crack changes its speed for the first time, the calculation such as proceeded in the present paper is facilitated by the property of self-similarity, as observed in Broberg (1999b). Even for this case, the analytical result obtained is rather intricate so that the numerical evaluation has to be invoked to observe its property. After the first change of speed, the elastic field is no longer self-similar. That would lead to a more difficult, if not impossible, process to obtain a solution of analytical feature.

Acknowledgements

W.Y. and Y.H. acknowledge the support from NSFC for the joint research collaboration grant between them on intersonic fracture investigation. Y.H. and A.J.R. acknowledge the support from ONR (#N00014-01-1-0205 and #N00014-95-1-0453, respectively, under program monitor Dr. Y.D.S. Rajapakse).

References

Abraham, F.F., Gao, H., 2000. How fast can cracks propagate? Phys. Rev. Lett. 84, 3113–3116.

Andrews, D.J., 1976. Rupture velocity of plane strain shear cracks. J. Geophys. Res. 81, 5679–5687.

Antipov, Y.A., Willis, J.R., 2002. Transient loading of a rapidly-advancing crack in a viscoelastic medium. Mech. Mater., in press.

Archuleta, R.J., 1982. Analysis of near source static and dynamic measurements from the 1979 imperial valley earthquake. Bull. Seismol. Soc. Am. 72, 1927–1956.

Beroza, G.C., Spudich, P., 1988. Linearized inversion for fault rupture behavior—application to the 1984 Morgan-Hill, California, earthquake. J. Geophys. Res. 93, 6275–6296.

Broberg, K.B., 1989. The near-tip field at high crack velocities. Int. J. Fract. 39, 1–13.

Broberg, K.B., 1994. Intersonic bilateral slip. Geophys. J. Int. 119, 706–714.

Broberg, K.B., 1999a. Intersonic crack propagation in an orthotropic material. Int. J. Fract. 99, 1–11.

Broberg, K.B., 1999b. Intersonic mode II crack acceleration. Fatigue Fract. Eng. Mater. Struct. 22, 17–24.

Brock, L.M., 1977. Two basic problems of plane crack extension: a unified treatment. Int. J. Eng. Sci. 15, 527–536.

Burridge, R., 1973. Admissible speeds for plane strain shear cracks with friction but lacking cohesion. Geophys. J. Roy. Soc. London 35, 439–455.

Burridge, R., Conn, G., Freund, L.B., 1979. The stability of a rapid mode II shear crack with finite cohesive traction. J. Geophys. Res. 85, 2210–2222.

Coker, D., Rosakis, A.J., 2001. Experimental observations of intersonic crack growth in assymmetrically loaded unidirectional composite plates. Philos. Mag. A 81 (3), 571–595.

Ellsworth, W.L., Celebi, M., 1999. Near field displacement time histories of the M 7.4 Kocaeli (Izimit), Turkey, earthquake of August 17, 1999. EOS Trans. Am. Geophys. Union 80 (46), F648.

Freund, L.B., 1972. Crack propagation in an elastic solid subjected to general loading. II. Nonuniform rate of extension. J. Mech. Phys. Solids 20, 141–152.

Freund, L.B., 1979. The mechanics of dynamics shear crack propagation. J. Geophys. Res. 84, 2199–2209.

Freund, L.B., 1990. Dynamic Fracture Mechanics. Cambridge University Press, Cambridge, UK.

Gao, H., Huang, Y., Gumbsch, P., Rosakis, A.J., 1999. On radiation-free transonic motion of cracks and dislocations. J. Mech. Phys. Solids 47, 1941–1961.

Gao, H., Huang, Y., Abraham, F., 2001. Continuum and atomistic studies of intersonic crack propagation. J. Mech. Phys. Solids 49, 2113–2132.

Geubelle, P.H., Kubair, D., 2001. Intersonic crack propagation in homogeneous media under shear dominated loading: numerical analysis. J. Mech. Phys. Solids 49, 571–687.

Huang, Y., Gao, H., 2001. Intersonic crack propagation. Part I: the fundamental solution. J. Appl. Mech. 68, 169–175.

Huang, Y., Gao, H., 2002. Intersonic crack propagation. Part II: suddenly Stopping Crack. J. Appl. Mech. 69, 76–80.

Huang, Y., Wang, W., Liu, C., Rosakis, A.J., 1999. Analysis of intersonic crack growth in unidirectional fiber-reinforced composites. J. Mech. Phys. Solids 47, 1893–1916.

Kostrov, B.V., 1975. On the crack propagation with variable velocity. Int. J. Fract. 11, 47–56.

Needleman, A., 1999. An analysis of intersonic crack growth under shear loading. ASME J. Appl. Mech. 66, 847–857.

Needleman, A., Rosakis, A.J., 1999. The effect of bond strength and loading rate on the conditions governing the attainment of intersonic crack growth along interfaces. J. Mech. Phys. Solids 47, 2411–2449.

Rosakis, A.J., Samudrala, O., Coker, D., 1999. Cracks faster than the shear wave speed. Science 284, 1337–1340.

Rosakis, A.J., Samudrala, O., Coker, D., 2000. Intersonic shear crack growth along weak planes. Mater. Res. Innovation 3, 236–243.

Simonov, I.V., 1983. Behavior of solutions of dynamic problems in the neighborhood of the edge of a cut moving at transonic speed in an elastic medium. Mech. Solids 18, 100–106.

Wald, D.J., Heaton, T.H., 1994. Spatial and temporal distribution of slip for the 1992 Landers, California, earthquake. Bull. Seismol. Soc. Am. 84, 668–691.

Willis, J.R., 1989. Accelerating cracks and related problems. In: Ogden, R.W. (Ed.), Elasticity, Mathematical Methods and Applications, The Ian Sneddòn 70th Birthday Volume. Ellis Horwood, Chichester, pp. 397–409.

Yu, H.H., Suo, Z., 2000. Intersonic crack growth on an interface. Proc. R. Soc. London Ser. A A456, 223–246.